Nalanda Open University

Annual Examination - 2018

B.Sc. Mathematics (Honours), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

Answer any ***Five*** *questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) Define the Cartesian product of two non-empty sets A and B and prove that if A, B, C are any three non--empty sets then
$A \times (B - C) = A \times B - A \times C$ and
$A \times (B \cap C) = (A \times B) \cap (A \times C)$
(b) Define on equivalence relation on a non-empty set A and if R_1 and R_2 are any two equivalence relations on A then show that $R_1 \cap R_2$ is also an equivalence relation on A.

2. (a) Show that a countable union of countable sets is countable.
(b) Show that the set R of all real numbers is uncountable.

3. (a) What do you mean by a denumerable set. Prove that every infinite set has a denumerable subset.
(b) If $f : X \to Y$ and $A \subseteq X$, $B \subseteq X$ then show that $f(A \cap B) \subseteq f(A) \cap f(B)$.

4. (a) What do you mean by a partially ordered set. If X is any non-empty set then show that $(P(X), \subseteq)$ is a partially ordered set.
(b) What do you mean by a Lattice and a complete Lattice, Give one example of each.

Group - B

5. Show that $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I = 0$. Hence or otherwise find the inverse of A.

6. Find the rank of the matrix $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$.

7. (a) Prove that $A(\text{adj } A) = (\text{adj } A)A = |A|$ where A is n-rowed square matrix.
(b) Find the eigen values of the matrix. $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$.

8. Solve by matrix method the following simultaneous equations.
$x + y + z = 6$ $2x + y - 3z = -5$ $3x - 2y + z = 2$

Group - C

9. (a) State and prove Lagranges Theorem.
(b) Prove that the intersection of two subgroups of a group G is also a subgroup of that group.

10. (a) Prove that if for every element 'a' in a group G, $a^2 = e$ then G is an abelian group.
(b) Prove that any two left or right cosests of a subgroup of a group G are either disjoint or identical.

11. (a) The equation $3x^4 - 25x^3 + 50x^2 - 50x + 12 = 0$ has two roots whose product is $2i$, find all the roots.
(b) Find the condition that the cubic $x^3 - px^2 + qx + r = 0$ should have its roots be in Harmonic progression.

12. (a) Find the expansion of $\sin\theta$ is ascending powers of θ.
(b) State and prove Gregories series for expansion of $\tan^{-1}x$ in ascending powers of x.

Nalanda Open University
Annual Examination - 2018
B.Sc. Mathematics (Honours), Part-I
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) State and prove Leibnitz's theorem.
   (b) If $y = \sin(m \sin^{-1} x)$ then prove that :
   $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$

2. (a) State and prove Maclaurin's theorem.
   (b) Prove that $\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \ldots\ldots\ldots \infty$

3. Evaluate the following limits.
   (a) $\underset{x \to 0}{Lt}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$ (b) $\underset{x \to \frac{\pi}{2}}{Lt}(\sin x)^{\tan x}$.

4. (a) State and prove Euler's theorem for homogeneous functions in two independent variables x and y and of degree n.
   (b) If $v = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \sin 2v.$

5. (a) Show that the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
   touches the curve $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$.
   (b) Find the pedal equation of the curve $r^n = a^n \sin(n\theta)$

Group - B

6. Evaluate any two of the following:
   (a) $\int \frac{d\theta}{(a^2\sin^2\theta + b^2\cos^2\theta)^2}$ (b) $\int \sqrt{\sec x + 1}$ (c) $\int (\sqrt{\tan x} + \sqrt{\cot x})dx$

7. Evaluate :
   (a) $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ (b) $\int_0^\pi x\log(\sin x)dx$

8. (a) If $I_{m,n} = \int \cos^m x \sin^n x\, dx$ then show that $(m+n)\, I_{m,n} = \cos^{m-1} x \cdot \sin^{n+1} x + (m-1)I_{m-2,n}$
   (b) Evaluate $\underset{r\to\infty}{lt}\sum_{r=1}^{n}\frac{r^3}{r^4+n^4}$.

9. Find the area between the curve $x(x^2+y^2) = a(x^2-y^2)$ and its asymptote. Also find the area of the loop.

10. Find the volume formed by the revolution of the loop of the curve $y^2(a-x) = x^2(a+x)$

Group – C

11. (a) Find the angle between two lines whose direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) are given.
    (b) Find the equation of the plane cutting off intercepts a, b, c from the axes.

12. (a) Show that $3x^2 + 4y^2 + 5z^2 - 6yz - 4zx - 2xy = 0$ represents a pair of planes.
    (b) Prove that the spheres
    $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ cut each other orthogonally.
    or Find the equation of the normal at any point of the conic $\frac{l}{r} = 1 + e\cos\theta$.

Nalanda Open University

Annual Examination - 2018

B.Sc. Mathematics (Subsidiary), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting atleast **One** question from each group.*
All questions carry equals marks.

Group - A

1. (a) If $f: X \rightarrow Y$ and $A \subseteq X, B \subseteq X$ then prove that:
   $f(A \cap B) \subseteq f(A) \cap f(B)$ and equality holds if f is one-one.
   (b) What do you mean by an equivalence relation on a set A. If R_1 and R_2 are two equivalence relations on A then show that $R_1 \cap R_2$ is also an equivalence relation on A.

2. (a) Define the Cartesian product of two non-empty sets A and B. If A, B, C are three non-empty sets then prove that $(A - B) \times C = A \times C - B \times C$.
   (b) Define Reflexive, Symmetric and Transitive relations giving one example of each.

3. (a) If G is group then prove that $(ab)^{-1} = b^{-1}a^{-1}\ \forall\ a, b \in$ G.
   (b) If a and b are any two elements of a group G then prove that the equation $ax = b$ and $ya = b$ have unique solution in G.

4. Prove that the set Pn of all permutations on n symbols is a finite non-abelian group of order n with respect to composition of mappings as the operation.

Group - B

5. State and prove De-Moivre's theorem.

6. (a) Reduce $(\alpha + i\beta)^{x + iy}$ in the form of $A + iB$.
   (b) Find $(1+i)^{\frac{1}{3}}$?

7. (a) Show that the sequence (x_n) where
   $x_1 = 1, x_n = \sqrt{2 + x_{n-1}}$ is convergent and it converges to 2.
   (b) Prove that every convergent sequence is bounded.

8. (a) Prove that a monotonic increasing sequence which is bounded above is convergent.
   (b) Prove that the sequence whose n^{th} term is $\left(\sqrt{n+1} - \sqrt{n}\right)$ is convergent.

Group - C

9. (a) If $y = \tan^{-1}x$ then prove that:
   $(1 + x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$
   (b) Apply Maclaurin's series to expand $e^{x \sec x}$ as for as the term containing x^3.

10. Evaluate :
    (a) $\underset{x \to 0}{Lt} \dfrac{x \cos x - \log(1+x)}{x^2}$ (b) $\underset{x \to 0}{Lt} (\cot x)^{\left(\frac{1}{\log x}\right)}$

11. (a) Give the geometrical meaning of scalar triple product of three vectors $\vec{a}, \vec{b}, \vec{c}$.
    (b) Prove that : $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$

12. (a) Prove that the two spheres:
    $S_1 \equiv x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and
    $S_2 \equiv x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$
    cut each other orthogonally if $2(u_1u_2 + v_1v_2 + w_1w_2) = d_1 + d_2$.
    (b) If the point $(at_1^2, 2at_1)$ is one extremity of a focal chord of the parabola $y^2 = 4ax$ then find the co-ordinates of the other extremity and hence show that the length of the chord is $a\left(t_1 + \dfrac{1}{t_1}\right)^2$.

Nalanda Open University

Annual Examination - 2018

B.Sc. Mathematics (Honours), Part-II, Paper-III

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.
All questions carry equal marks.

Group-A

1. (a) State and prove Bolzano Weierstrass theorem.
   (b) State and prove Heine-Borel theorem.
2. (a) Prove that every compact subset of R is closed.
   (b) Prove that every closed subset of a compact set in R is compact.
3. (a) Prove that a set E in R is compact if and only if E is closed and bounded.
   (b) Prove that Int. (A) is an open set.

Group-B

4. (a) Prove that every Cauchy Sequence of real numbers is convergent.
   (b) If (x_n) is a sequence where $x_n = \left(\sqrt{n+1} - \sqrt{n}\right)$ for all $n \in N$ then show that it is convergent and find its limit.
5. (a) Prove that every bounded monotonically increasing sequence converges to its least upper bound.
   (b) Prove that every monotonically decreasing sequence which is bounded tends to its greatest lower bound.
6. (a) Test the convergence of the series $\sum \frac{1}{n^p}$.
   (b) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$.
7. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n}$.
   (b) State and prove Logarithmic ratio test.

Group-C

8. Prove that $T : V_2(R) \rightarrow V_3(R)$ defined by $T(a, b) = ((a + b), (a - b), b)$ is a linear transformation.
9. If W_1 and W_2 are two sub spaces of a finite dimensional vector space Vover a field F then show that

   $$\dim(W_1 + W_2) = \dim \cdot W_1 + \dim \cdot W_2 - \dim \cdot (W_1 \cap W_2).$$

10. (a) Define the eigen values and eigen vectors of a square matrix and compute the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
    (b) Prove that the set $(1, i, o), (2i, 1, 1), (1, 1 + i, 1 - i)$ is a basis for $V_3(C)$.

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Examination Programme, 2018

(Bachelor of Science, Mathematics (Honours), Part-II

Date	Paper	Time	Name of Examination Centre
26/2/2018	HONOURS PAPER – III	**12.00 to 3.00 pm**	Nalanda Open University, Patna
28/2/2018	HONOURS PAPER – IV	**12.00 to 3.00 pm**	Nalanda Open University, Patna
05/3/2018	(SUB.) (Botany - II)	8.00 to 11.00 am	Nalanda Open University, Patna
06/3/2018	Hindi 100 or Ur 50+Hn 50	**3.30 to 6.30 pm**	Nalanda Open University, Patna
07/3/2018	(SUB.) (Mathematics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
09/3/2018	(SUB.) (Chemistry - II)	8.00 to 11.00 am	Nalanda Open University, Patna
10/3/2018	(SUB.) (Physics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
12/3/2018	(SUB.) (Zoology - II)	8.00 to 11.00 am	Nalanda Open University, Patna
14/3/2018	(SUB.) (Geography - II)	8.00 to 11.00 am	Nalanda Open University, Patna
16/3/2018	(SUB.) (Home Science- II)	8.00 to 11.00 am	Nalanda Open University, Patna
19/3/2018	(SUB) Statistics-II)	8.00 to 11.00 am	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2018

B.Sc. Mathematics (Honours), Part-II,

Paper-IV

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group. All questions carry equal marks.

Group-A

1. Solve : (a) $p(p + x) = y(x + y)$ (b) $y = (1+ p)x + ap^2$.

2. (a) Obtain the primitive and singular solution of the equation $xp^2 - 2yp + 4x = 0$.

   (b) Solve the differential equation $(8p^3 - 27)x = 12p^2y$ and investigate whether a singular solution exists.

3. (a) Solve : $\dfrac{d^2y}{dx^2} + \cot x \dfrac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$.

   (b) Solve by the method of variation of parameters the equation : $\dfrac{d^2y}{dx^2} + a^2 y = \operatorname{cosec} ax$.

Group-B

4. (a) Find the volume of the parallelopiped whose edges are represented by :

   $\vec{i} + \vec{j} + \vec{k},\ \vec{i} - \vec{j} + \vec{k},\ \vec{i} + 2\vec{j} - \vec{k}$.

   (b) Prove that : $[\vec{a}\ \vec{b}\ \vec{c}][\vec{p}\ \vec{q}\ \vec{r}] = \begin{bmatrix} \vec{a}\cdot\vec{p} & \vec{b}\cdot\vec{p} & \vec{c}\cdot\vec{p} \\ \vec{a}\cdot\vec{q} & \vec{b}\cdot\vec{q} & \vec{c}\cdot\vec{q} \\ \vec{a}\cdot\vec{r} & \vec{b}\cdot\vec{r} & \vec{c}\cdot\vec{r} \end{bmatrix}$.

5. (a) Prove that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$.

   (b) Evaluate : $\dfrac{d^2}{dt^2}\left\{\left(\vec{r}\times\dfrac{d\vec{r}}{dt}\right)\times\dfrac{d^2\vec{r}}{dt^2}\right\}$.

6. (a) Prove the $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot(\nabla\times\vec{u}) - \vec{u}\cdot(\nabla\times\vec{v})$.

   (b) Prove that $\nabla\cdot(\nabla\times\vec{u}) = 0$ or div. curl $\vec{u} = 0$.

7. (a) If $\vec{a}$ and $\vec{b}$ are constant vectors and $\vec{r} = (x, y, z)$, then prove that :

   $\nabla\cdot\left\{\vec{a}\times\left(\nabla\left(\dfrac{1}{\vec{r}}\right)\right)\right\} = 0.$

   (b) Find the unit normal vector to the level surface $x^2 + y - z = 4$ at the point (2, 0, 0).

Group-C

8. State and prove the principle of virtual work.

9. In a simple Harmonic motion if u, v, w be velocities at distances a, b, c respectively from a fixed point on a straight line which is not the centre of force, then show that the periodic time T is given by the equation.

   $$\frac{4\pi^2}{T}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$

10. (a) Show that the modulus of an elastic string is equal to the force which would stretch a light string to twice its natural length.

    (b) What are the forces that can be neglected during forming the equation of virtual work.

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Nalanda Open University
Annual Examination - 2018
B.Sc. Mathematics (Subsidiary), Part-II,
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Eight questions, selecting atleast one from each group. All questions carry equal marks.

Group-A

1. Evaluate any two of the following integrals:

 (a) $\int \frac{x^2 dx}{(1-x^4)\sqrt{1+x^4}}$ (b) $\int \frac{dx}{\sqrt{(x-a)(x-b)}}$ (c) $\int \frac{2x+3}{\sqrt{x^2+x+1}} dx$

2. Evaluate any two of the following definite integrals:

 (a) $\int_0^{\pi} \frac{x\, dx}{1+\sin x}$ (b) $\int_0^{a} \frac{dx}{a+b\cos x}$ (c) $\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x + b^2 \sin^2 x}$

3. (a) Evaluate $\underset{n\to\infty}{\text{limit}}\left[\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots\dots\dots+\frac{n}{n^2+(n-1)^2}\right]$.

 (b) Obtain a reduction formula for $\int \sin^m x \cos^n x\, dx$.

4. Find the area of the loop of the curve
 $y^2(a^2+x^2) = x^2(a^2-x^2)$
5. Find the perimeter of the loop of the curve
 $9ay^2 = (x-2a)(x-5a)^2$
6. Find the length of one quadrant of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.
7. Find the surface of the solid of revolution by revolving the curve $r^2 = a^2\cos 2\theta$.
8. Find the volume of the spindle shaped solid generated by revolving asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about x-axis.
9. Solve the following Differential equations:

 (a) $p(p+x) = y(x+y)$ (b) $y = x\left\{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2\right\}$.

10. Solve the following Differential equations:

 (a) $\frac{d^2y}{dx^2} + a^2 y = \sec ax$ (b) $\frac{d^2y}{dx^2} - y = x\sin x + (1+x^2)e^x$..

Group-B

11. Find the equation of the right circular cylinder which passes through the circle
 $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$.
12. Find the equation of the sphere which passes through (α, β, γ) and the circle
 $x^2 + y^2 + z^2 = a^2$, $z = 0$.
13. (a) Define a convex set $S \subseteq R^2$ and prove that the sphere is a convex set.
 (b) Prove that a hyper plane is a closed set.

Group-C

14. Find the equation of line of action of co-planar forces and its resultant.
15. If forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x \cos\alpha + y \sin\alpha = p$. Find the magnitude of the resultant and its line of action.
16. Define simple Harmonic Motion and show that how two simple Harmonic motions can be compounded in a straight line.

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Nalanda Open University
Annual Examination - 2018
B.Sc. Mathematics (Honours), Part-III
Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) State and prove Minkowsky's inequality.
   (b) State and prove Cauchy's Schwartz inequality.
2. (a) In a metric space (X, d), prove that the union of an arbitrary collection of open sets is open.
   (b) In a metric space (X, d), prove that any finite intersection of open sets in X is open.
3. (a) Define the convergence of a sequence (x_n) in a metric space (X, d) and prove that the limit of a sequence in (X, d) if it exists is unique.
   (b) Define a Cauchy Sequence in a metric space (X, d) and prove that every convergent sequence in (X, d) is a Cauchy sequence in (X, d).
4. Prove that (R^n, d) is complete where d on R^n is defined as $d(x, y) = \left[\sum_{2=1}^{n} |x_i - y_i|^2\right]^{\frac{1}{2}}$.
5. (a) Let (X, d) be a metric space and $A \subseteq X$ then show that A is closed if and only if $A \subseteq \overline{A}$.
   (b) If M and N are two subsets of a metric space (X, d) then show that $\overline{MUN} = \overline{M} \cup \overline{N}$.

Group 'B'

6. Let (X, T) be a Topological space and A and B are any two subsets of X. If $\overline{A}$ denotes the closure of A then prove that :
   (a) $\overline{\phi} = \phi$ (b) $A \subseteq \overline{A}$ (c) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$
   (d) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (e) $\overline{(A \cap B)} \subseteq \overline{A} \cap \overline{B}$ (f) $\overline{\overline{A}} = A$
7. (a) Let (X, T_1) and (Y, T_2) be two Topological spaces then a function $f : X \to Y$ is $T_1 \to T_2$ continuous if and only if for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.
   (b) Let (X, T_1) and (Y, T_2) be two Topological spaces then a mapping $f : X \to Y$ is open if and only if $f(A^{\circ}) \subseteq [f(A)]^{\circ}$ for every subset of X.

Group 'C'

8. (a) If a function f is continuous on $[a, b]$ then prove that it is Riemann integrable on $[a, b]$.
   (b) Prove that every bounded monotonic function $f : [a, b] \to R$ is R-integrable on [a, b].
9. (a) If f and g are bounded and R-integrable in $[a, b]$ then prove that $f + g$ is also bounded and R-integrable in $[a, b]$ and $\int_a^b \{f(x) \pm g(x)\}dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$.
   (b) If found g are two bounded and R-integrable functions in $[a, b]$ then prove that fg is bounded and R-integrable in $[a, b]$.

Group 'D'

10. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n \log n (\log \log n)^p}$.
11. (a) Find the radius of convergence of the series $\sum \frac{n^n x^n}{n}$.
    (b) Prove that the series $\sum \left(\frac{\cos n\theta}{n^2}\right)$ is convergent for all real values of θ.

Examination Programme-2018
B.Sc (Part–III) Botany, Chemistry, Mathematics, Physics & Zoology Honours

Date	Papers	Time	Examination Centre
15/2/2018	Honours Paper–V	8 to 11 AM	Nalanda Open University, Patna
17/2/2018	Honours Paper–VI	8 to 11 AM	Nalanda Open University, Patna
19/2/2018	Honours Paper–VII	8 to 11 AM	Nalanda Open University, Patna
21/2/2018	Honours Paper–VIII	8 to 11 AM	Nalanda Open University, Patna
23/2/2018	Paper –XV (General Studies)	8 to 11 AM	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2018

B.Sc. Mathematics (Honours), Part-III

Paper-VI

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Prove that if a group G has four elements then it must be abelian, group.
   (b) Prove that the order of an element a of a group G is equal to the order of $f(a)$.
2. (a) State and prove Lagrange's Theorem.
   (b) Prove that the order of every element of a finite group is a divisor of the order of the group.
3. (a) Sate and prove Caley's Theorem.
   (b) Prove that evry group is isomorphic to a group of one-one onto functions.

Group 'B'

4. (a) Define a normal subgroup of a group G. Show that every subgroup of an abelian group is normal.
   (b) If f is a homomorphism of a group G into a group G'. Then prove that the Kernel K of G is a normal subgroup of G.
5. (a) If R is a commutative ring with unity element then show that R is a field if and only if it has non-trivial ideals.
   (b) Let $f(x) = x^4 + x^3 - 3x^2 - x + 2$ and $g(x) = x^4 + x^3 - x^2 + x - 2$
   Then find the g.c.d. of $f(x)$ and $g(x)$ as polynomials over Q.

Group 'C'

6. (a) State and prove Zorn's Lemma.
   (b) Prove that $2^{No} = c$.
7. (a) State and prove Schroder-Bernstein Theorem.
   (b) Show that the set of all real numbers in [0, 1] is not denumerable.
8. (a) Prove that $N \times N$ is countable.
   (b) If A_i is countably infinite set then prove that $\bigcup_{i=1}^{\infty} A_i$ is countably infinite set.

Group 'D'

9. (a) If $f(z) = u + iv$ is analytic function and $u - v = e^x(\cos y - \sin y)$ find $f(z)$ in terms of z.
   (b) Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.
10. (a) Find the radius of convergence of the series $\frac{z}{2} + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^3 + \ldots\ldots\ldots\infty$.
    (b) Find the domain of the convergence of the series $\sum_{n=1}^{\infty} \frac{1.3.5\ldots\ldots\ldots(2n-1)}{n\,!}\left(\frac{1-z}{z}\right)^n$.
11. State and prove Cauchy integral formula.

Examination Programme-2018

B.Sc (Part–III) Botany, Chemistry, Mathematics, Physics & Zoology Honours

Date	Papers	Time	Examination Centre
15/2/2018	Honours Paper–V	8 to 11 AM	Nalanda Open University, Patna
17/2/2018	Honours Paper–VI	8 to 11 AM	Nalanda Open University, Patna
19/2/2018	Honours Paper–VII	8 to 11 AM	Nalanda Open University, Patna
21/2/2018	Honours Paper–VIII	8 to 11 AM	Nalanda Open University, Patna
23/2/2018	Paper –XV (General Studies)	8 to 11 AM	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2018

B.Sc. Mathematics (Honours), Part-III

(Graphpaper may be supplied)

Paper-VII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Prove that a sphere is a convex set.
   (b) Prove that the set of all feasible solutions of a linear programming problem constitutes a convex set.
2. (a) Minimize $z = x_1 - 3x_2 + 2x_3$ subject to the conditions.
   $3x_1 - x_2 + 2x_3 \leq 7$
   $-2x_1 + 4x_2 \leq 12$
   $-4x_1 + 3x_2 + 8x_3 \leq 10$
   $x_1, x_2, x_3 \geq 0$
   (b) Maximize $z = 3x + 5y + 4z$. Subject to the conditions.
   $2x + 3y \leq 8$
   $2y + 5z \leq 10$
   $3x + 2y + 4z \leq 15$
   $x, y, z \geq 0.$
3. Solve the following L.P.P. problem by simplex method.
   Maximize $z = 4x_1 + 10x_2$. Subject to the conditions.
   $2x_1 + x_2 \leq 50$
   $2x_1 + 5x_2 \leq 100$
   $2x_1 + 3x_2 \leq 90$
   $x_1, x_2 \geq 0.$

Group 'B'

4. (a) Solve $\dfrac{dx}{x^2 - yz} = \dfrac{dy}{y^2 - zx} = \dfrac{dz}{z^2 - xy}$.
   (b) Solve $(2xz - yz)dx + (2yz - zx)dy - (x^2 - xy + z^2)dz = 0$
5. (a) Solve $\dfrac{dx}{dt} + 4x + 3y = t^2$ and $\dfrac{dy}{dt} + 2x + 5y = e^{2t}$.
   (b) Solve $t\dfrac{dx}{dt} + y = 0$ and $t\dfrac{dy}{dt} + x = 0$.
6. (a) Solve $(y^2 + z^2 = x^2)p - 2xyq + 2zx = 0$.
   (b) Solve $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$
7. Use Charpit's method to solve :
   (a) $(p^2 + q^2)y = qz$. (b) $pxy + pq + qy - yz = 0$.
8. (a) Solve $r = a^2t$ by Monge's method.
   (b) Solve $r = t\cos^2 x + p\tan x = 0$ by Monge's method.

Group 'C'

9. (a) Find the attraction of a circular disc at an external point at height h.
   (b) Find the potential of a circular disc at a point distant h on the axis from the centre.
10. Find the centre of pressure of a vertical circle of radius a wholly immersed in a homogeneous liquid with its centre at a depth h below the free surface.

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Annual Examination - 2018

B.Sc. Mathematics (Honours), Part-III

Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Five questions. All questions carry equal marks.

1. (a) Express the following functions and their differences in the factorial notation.

   (i) $y = 2x^3 - 3x^2 + 3x - 10$ (ii) $y = x^4 - 12x^3 + 42x^2 - 30x + 9$.

   (b) If $f(x)$ and $g(x)$ are any functions of x then prove that :

   (i) $\Delta\left[f(x)\,g(x)\right] = f(x)\Delta\,g(x) + \Delta g(x+1)\Delta f(x) = f(x+1)\Delta\,g(x) + g(x)\,\Delta f(x)$

   (ii) $\Delta\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)\,g(x+1)}$.

2. (a) Show that if Δ operates on n, then :

   $\Delta\begin{pmatrix} n \\ x+1 \end{pmatrix} = \begin{pmatrix} n \\ x \end{pmatrix}$ and hence deduce that $\sum_{n=1}^{N}\begin{pmatrix} n \\ x \end{pmatrix} = \begin{pmatrix} N+1 \\ x+1 \end{pmatrix} - \begin{pmatrix} 1 \\ x+1 \end{pmatrix}$.

   (b) Prove that : $\cup_x - \cup_{x+1} + \cup_{x+2} - \cup_{x+3} + \ldots\ldots\ldots\ldots$

   $= \dfrac{1}{2}\left[\cup_{x-\frac{1}{2}} - \dfrac{1}{2}\Delta^2 \cup_{x-\frac{3}{2}} + \dfrac{1.3}{2!8^2}\Delta^4 \cup_{x-\frac{5}{2}} + \ldots\ldots\ldots\right]$

3. (a) Show that if n is a positive integer then : $(x\Delta)^n \cup_x = (x+n-1)^{(n)}\,\Delta^n \cup_x$.

   (b) Prove that : $\cup_1 x + \cup_2 x^2 + \cup_3 x^3 + \ldots\ldots\ldots$

   $= \dfrac{x}{1-x}\cup_1 + \dfrac{x^2}{(1-x)^2}\Delta\cup_1 + \dfrac{x^3}{(1-x)^3}\Delta^2\cup_1 + \ldots\ldots\ldots$

4. (a) Estimate the missing figure in the following table :

x :	1	2	3	4	5
$f(x)$:	2	5	7	X	32

   (b) Find the sixth term of the series : 8 + 12 + 19 + 29 + 42 + ...

5. (a) Prove that : $\Delta^n O^{n+1} = \dfrac{n(n+1)}{2}\Delta^n O^n$.

   (b) Prove that : $\dfrac{\Delta^n O^m}{\lfloor n} = \dfrac{n\Delta^n O^{m-1}}{\lfloor n} + \dfrac{\Delta^{n-1} O^{m-1}}{\lfloor n-1}$.

6. (a) What is the form of the function of the following table.

x :	0	1	4	5
$f(x)$:	8	11	68	123

   (b) Find the polynomial of the lowest degree which assumes the values 3, 12, 15, –21. When x has the values 3, 2, 1, –1 respectively.

7. If $f(20) = 14$, $f(24) = 32$, $f(28) = 35$, $f(32) = 40$.

   Then by Gauss's forward formula show that $f(25) = 33 \cdot 49$.

8. Find the maximum and minimum values of the function tabulated below.

x :	0	1	2	3	4	5
$f(x)$:	0	0.25	0	2.25	16.00	56.25

9. Solve the equation $2 + \log_{10}^{x} = 2e^{-x}$ by the method of iteration.

10. (a) Evaluate $\int_0^6 \dfrac{dx}{1+x^2}$ using

    (i) Simpson's 1/3rd rule and (ii) Simpson's 3/8th rule.

    (b) Find the solution of the difference equation

    $u_{x+4} - 7u_{x+1} + 12u_x = \cos x$.