NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–I
(Advanced Abstract Algebra)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that any two sub-normal series of a group have equivalent refinements.
   (b) Prove that a finite group G-has a composition series.
2. (a) Prove that if G is a commutative group having a composition series then G is finite.
   (b) Find all the composition series of $Z_5 \times Z_5$.
3. (a) State and prove Unique Factorization theorem.
   (b) In the ring $Z \,|\, [12]$ find the H.C.F and L.C.M of [6] and [8].
4. (a) Define Modules and sub-Modules and prove that union of two sub-modules is a sub-module iff one is contained in the other.
   (b) If A and B are sub-modules of R-module M, then show that $\frac{A+B}{B} = \frac{A}{A \cap B}$.
5. (a) Prove that if M is a finite extension field of a field K and K is finite extension field of a field F, then M is a finite extension of F and $[M : F] = [M : K][K : F]$.
   (b) Prove that every finite extension of a field is an algebraic extension.
6. (a) State and prove Remainder theorem.
   (b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
7. (a) Prove that any finite extension of a field of characteristics zero is a simple extension.
   (b) If K be an extension of a field F and a be an algebraic element of odd degree over F then show that a^2 is algebraic over F and that $F(a) = F(a^2)$.
8. If K is a field and $\sigma_1, \sigma_2, \sigma_3, ---, \sigma_n$ are distinct automorphisms of K, then prove that it is possible to find elements $x_1, x_2, x_3, ---, x_n$ not all zero in K such that $x_1\sigma_1(u) + x_2\sigma_2(u) + x_3\sigma_3(u) + --- + x_n\sigma_n(u) = 0 \quad \forall\, u \in K$.
9. Find the Galois group of equation $x^3 - 2 = 0$ over the field Q of rational numbers.
10. Find the spliting field K of polynomial $x^4 - x^2 + 1$ over Q, the field of rational numbers. Also determine the Galois group of K over Q. Show that it not a cyclic group.

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Examination Programme, 2016

M.Sc. Mathematics, Part-I

Date	*Papers*	*Time*	*Examination Centre*
12.05.2016	Paper-I	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
14.05.2016	Paper-II	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
16.05.2016	Paper-III	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
18.05.2016	Paper-IV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
20.05.2016	Paper-V	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
24.05.2016	Paper-VI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
26.05.2016	Paper-VII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
28.05.2016	Paper-VIII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–II
(Real Analysis)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove Cantor's intersection theorem.
   (b) Prove that a subset F of R is closed if and only if F contains all its accumulation points.
2. (a) Prove that every function of bounded variation on [a, b] is necessarily bounded.
   (b) If *f* and *g* are functions of bounded variation on [a, b] then show that *fg* is also a function of bounded variation on [a, b].
3. (a) If *f* is continuous on [a, b] then prove that *f* is integrable with respect to α on [a, b] in the sense of Riemann-Stieltjes.
   (b) If *f* be bounded and α monotonically increasing on [a, b]. Let $c \in [a, b]$ then if the two integrals $\int_a^c f d\alpha, \int_c^b f d\alpha$ exist then $\int_a^b f d\alpha$ exists and $\int_a^b f d\alpha = \int_a^c f d\alpha + \int_c^b f d\alpha$.
4. (a) Prove that if f is continuous and α is monotonically increasing on [a, b] then there exists a point $x \in [a, b]$ such that $\int_a^b f d\alpha = f(x)[\alpha(b) - \alpha(a)]$.
   (M.V.T. for Stieltges integral)
   (b) If $f \in R(\alpha)$ on [a, b] and α is monotonically increasing on [a, b] then $f^2 \in R(\alpha)$. Prove it. Also if $g \in R(\alpha)$ then prove that $(f \cdot g) \in R(\alpha)$.
5. (a) Let E be an open subset of R^n and $\underline{f}$ be a vector valued functions $\underline{f} : E \to R^m$. If $\underline{f}$ is differentiable at a point $\underline{c}$ of E then prove that $\underline{f}$ is continuous.
   (b) Let $f : R^2 \to R$ defined by
   $$f(x, y) = (x^2 + y^2)\log(x^2 + y^2) \quad if \ f(x, y) \neq (0, 0)$$
   $$= 0 \quad if \ f(x, y) = (0, 0)$$
   then prove that $D_{12} f(0, 0) = D_{21} f(0, 0)$.
6. State and prove Taylor's formula for a function of *n* variables.
7. Find the radius of convergences of the following power series.
   (a) $\sum_{n=0}^{\infty} \frac{\lfloor 2n}{(\lfloor n)^2} x^n$ (b) $\sum_{n=0}^{\infty} \frac{\lfloor n}{n^n} x^n$
8. State and prove Inverse function theorem.
9. Find the Jacobiam $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ if $x = r \, Sin\,\theta \, Cos\,\phi$, $y = r \, Sin\,\theta \, Sin\,\phi$, $z = r \, Cos\,\theta$.
10. Find the extreme value of the function *f* given by $f(x, y) = xy(a - x - y)$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–III
(Measure Theory)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If E_1 and E_2 are two measurable sets then (i) $E_1 - E_2$ is measurable and (ii) if $E_1 \supseteq E_2$ and $m(E_2) < \infty$ then $m(E_1 - E_2) = m(E_1) - m(E_2)$. Prove it.

   (b) If (E_n) be a sequence of pair wise disjoint measurable sets then show that $m\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} m(E_n)$.

2. (a) If (E_n) is a sequence of measurable sets such that $E_1 \subseteq E_2 \subseteq E_3 \subseteq --- \subseteq E_n \subseteq E_{n+1} \subseteq ---$ and $E = \bigcup_{n=1}^{\infty} E_n$ then prove that $m(E) = \underset{n \to \infty}{lt}\, m(E_n)$.

   (b) Prove that the class of all measurable sets forms a σ-ring.

3. Give the full construction of Cantor's Ternary set and prove that it an uncountable set of measure zero.

4. (a) Prove that the class of all measurable functions is closed with respect to all analytic operations.

   (b) Give the necessary and sufficient condition for a function f to be measurable.

5. State and prove bounded convergence theorem.

6. State and prove Egoroff's theorem.

7. (a) Prove that $\int_{A \cup B} f = \int_A f + \int_B f$ where $A \cap B = \phi$ in the sense of Lebesgue.

   (b) Prove that

   $$\int_A f\, d\mu = \int_X I_A f\, d\mu$$

   where $A \subseteq X$ and I_A is t he indicator function of A.

8. (a) Define absolute continuity. Prove that if f is absolutely continuous on [a, b] then it is of bounded variation on [a, b].

   (b) If f is absolutely continuous on [a, b] and $f'(x) = 0$ almost every where in [a, b] then confirm that f is constant in [a, b].

9. State and prove Jordan's decomposition theorem.

10. State and prove Dominated convergence theorem.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–IV
(Topology)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Define the boundary of a set in a Topological space. Show that a set *A* of a Topological space *(X, T)* is open iff $b(A) = \overline{A} - A$.
   (b) Show that *A* is open if and only if *A* is disjoint from its boundary.

2. (a) Define metric topology and exhibit all conditions under metric law considered.
   (b) Define co-finite topology on a non-empty set and give an example of it.

3. (a) Define a derived set and show that
   (i) $A \subseteq B \Rightarrow D(A) \subseteq D(B)$ (ii) $D(A \cup B) = D(A) \cup D(B)$
   where *A* and *B* are subsets of a topological space *(X, T)* and *D(A)* is the derived set of *A*.
   (b) Prove that the intersection of any two topologies on a non empty set *X* is a Topology on *X*.

4. (a) Define a Hausdorff space and give an example of a non-Hausdorff space in which every convergent sequence has unique limit.
   (b) Let *A* and *B* be subsets of a topological space *X*, then show that $b(A \cup B) = b(A) \cup b(B)$.

5. (a) Show by an example that continuous mapping is not necessarily on open mapping.
   (b) Prove that the open rectangle in the Euclidean plane forms an open base.

6. (a) Show that the property of being a T_2-space is both hereditary and topological.
   (b) Define continuity on a topological space. Prove that the mapping *f* of topological space *(X, T)* into an indiscrete space *(Y, I)* is continuous.

7. (a) Show that every compact sub-space of real line is closed and bounded.
   (b) Show that every co-finite space is compact.

8. (a) Prove that a topological space is normal iff each neighbourhood of a closed set *F* contains the closure of some neighbourhood of *F*.
   (b) Discuss sequential continuity and its relation with continuity.

9. (a) Let *(X, T)* be a topological space and *A* be a connected sub space of X. Show that $\overline{A}$ the closure of *A* is connected.
   (b) Prove that any continuous image of a connected space is connected.

10. (a) Let $X = \{a, b, c, d\}$ and $T = \{\phi, X, \{b\}, \{b, c\}, \{b, c, d\}\}$. Then show that X is connected.
    (b) Let $X = \{a, b, c\}$ and $T = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Show that it is a T_0-space but not a T_1-space.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–V
(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that if $V(F)$ is an n-dimensional space then V is isomorphic to F^n.
   (b) If $V_3(R)$ be a vector space and B = {(1, –1, 3), (0, 1, –1), (0, 3, –2)} be a basis of $V_3(R)$ then find its dual basis.

2. If $V = W_1 \oplus W_2 \oplus W_3 \oplus --- \oplus W_k$ then show that there exist linear operators $E_1, E_2, E_3, ----, E_k$ on V such that (i) Each E_i is a projection (ii) $E_i E_j = 0$ if $i \neq j$ (iii) $E_1 + E_2 + --- + E_k = I$ (iv) Range $E_i = W_i$.

3. Define a linear functional from a vector space to its field. Prove that a function f on R^n defined by $f(x) = f(x_1, x_2, ---, x_n) = a_1x_1 + a_2x_2 + --- + a_nx_n$ is a functional on R^n where $a_1, a_2, ---, a_n$ be fixed scalars in R.

4. If V be a finite dimensional vector space and B be a basis of V and B' be dual basis for V then show that $B'' = (B')' = B$.

5. (a) Define a bilinear form on a vector space $V(K)$. Show that $b(x, y) = [x_1, x_2]\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ is a real bilinear form.
   (b) Prove that two real biquadratic forms are equivalent iff they have the same rank and same index.

6. (a) Define Lattice and sub-lattice and make out their difference with examples.
   (b) What is a modular lattice ? Prove that the set L of all ideals of a ring is a modular lattice.

7. Define last upper bound and greatest lower bound on a partially ordered set $(X, \subseteq)$. Prove that the partially ordered set $(P(X), \subseteq)$ is a lattice.

8. State and establish sylvester's law of inertia.

9. (a) Define a Boolean algebra and give an example of it.
   (b) Prove that the intersection of any two sub-algebra of a Boolean-algebra B is a Boolean sub-algebra of B.

10. (a) Prove that if B is a Boolean-algebra and x, y, z $\in$ B then $x \wedge (y - z) = (x \wedge y) - (x \wedge z)$.
    (b) Reduce the matrix $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$ to Jordan canonical form.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VI
(Complex Analysis)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Derive necessary conditions for a function to be analytic in Cartesian form.
   (b) If $w = f(z) = u + i\upsilon$ and $u - i\upsilon = e^{x}\left(Cosy - Siny\right)$ then find w as a function of z.

2. What is radius of convergence of a power series ? Discuss the three possible cases for the vanishing of radius of convergence.

3. (a) Explain a bilinear transformation and describe its critical points in different situations.
   (b) Find the Mobius transformations which transforms unit circle $|z| \leq 1$ into unit cricle $|w| \leq 1$.

4. Show that continuity is necessary condition for differentiability but not sufficient. How a function can be enriched so as to become differentiable.

5. (a) State and prove Liouiville's theorem.
   (b) Evaluate $\int_c \frac{dz}{(z + \pi i)}$ where c is the circle $|z + 3i| = 1$.

6. (a) State and establish Cauchy integral formula.
   (b) Use nth derivative integral formula for deriving Cauchy's inequality.

7. State and prove Morera's theorem.

8. (a) State and prove Rouche's theorem.
   (b) Using residue theorem evaluate $\int_c \frac{e^{z}\,dz}{z(z-1)^2}$ where c is the circle $|z| = 2$.

9. Find the Laurent's expansion of $\frac{z}{(z+1)(z+2)}$ about singularity $z = -2$ and specify the region of convergence and nature of singularity at $z = -2$.

10. (a) Evaluate $\int_c \frac{e^{2z}\,dz}{(z+1)^2}$ where c is the circle $|z| = 3$.
    (b) What kind of singularities have the following functions
    (i) $\frac{1}{Sin\,z - Cos\,z}$ at $z = \pi/4$
    (ii) $Sin\,z - Cos\,z$ at $z = \infty$
    (iii) $\frac{1 - e^{z}}{1 + e^{z}}$ at $z = \infty$
    logically justify your answer.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–VII

(Theory of Differential Equations)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove Cauchy-Peano Existence theorem.

2. State and prove Ascolli's Lemma.

3. Define the linear system and show that it satisfies Lipschitz condition and set of solutions form a vector space.

4. Let $A(x)$ be continuous function on [a, b]. Then prove that the initial value problem (I.V.P) $\frac{dy}{dx} = A(x)y,\ y(r) = s,\ a \le r \le b,\ |s| < \infty$ has unique solution on [a, b].

5. Solve $y_1' = 2y_1 + y_2$ and $y_2' = 3y_1 + 4y_2$.

6. (a) Introduce the concept of e^A where A is a square matrix of order n and show that $|e^A| \le (n-1) + e^{|A|}$.

   (b) Find E^A where $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

7. Define fundamental matrix and show that a necessary and sufficient condition that a solution matrix G to be a fundamental matrix is that $G(x) \ne 0$ for $x \in I$.

8. Solve the system of differential equations by matrix method

   $$\frac{dx_1}{dt} = 9x_1 - 8x_2$$

   $$\frac{dx_2}{dt} = 24x_1 - 19x_2$$

   and initial conditions are $x_1(0) = 1,\ x_2(0) = 0$

9 What do you mean by critical points of a system. Find the nature of critical points (0, 0) of the system

   $$\frac{dx}{dt} = 2x + y$$

   $$\frac{dy}{dt} = 3x + 4y$$

10. Test the stability of the non-linear system

   $$\frac{dx}{dt} = x + 4y - x^2$$

   $$\frac{dy}{dt} = 6x - y + 2xy$$

   Also make a comment on the type of stability.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VIII
(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that the set of all real numbers is uncountable.
   (b) Prove that every set can be well ordered.
2. (a) For any cardinal numbers $\alpha,\ \beta,\ \gamma$; show that $\alpha^{\beta\gamma} = \left(\alpha^{\beta}\right)^{\gamma}$.
   (b) Prove that the axiom of choice implies Zorn's Lemma.
3. (a) Prove that the relation $V = E + R = 2$ where symbols have their usual meaning in graph theory and the graph G taken, is connected.
   (b) Show that a complete graph of n vertices is a planner if $n \leq 6$.
4. (a) Prove that an undirected graph is a tree-iff there is unique path between any two vertices.
   (b) If a tree has n vertices of degree 4. Find the value of n.
5. (a) State and prove Euler's theorem.
   (b) Express $\left(2^2 + 5^2\right)\left(3^2 + 8^2\right)\left(4^2 + 7^2\right)$ as sum of two squares.
6. (a) State and prove division algorithm in theory of numbers.
   (b) Find (24, 63) as a linear combination of 24 and 63.
7. (a) Define osculating plane and derive the vector and scalar equations for it at a point on the space curve.
   (b) For the curve defined as $\vec{r} = \left[a\left(3u - u^3\right), 3au^2, a\left(3u + u^3\right)\right]$. Prove that the curvature and torsion are equal.
8. (a) State and prove Chinese remainder theorem.
   (b) Find the general solution of $8x + 5y = 81$.
9. (a) Define un umblic. Prove that in general three lines of curvature pass through an umblic.
   (b) Show that for a geodesic $T^2 = \left(K - K_1\right)\left(K - K_2\right)$ where K is curvature and T is torsion of the geodesic.
10. (a) Explain associated Bertrand Curve and derive the result $TT_1 = \dfrac{Sin^2\alpha}{a^2}$ and $\left(1 - aR\right)\left(1 + aR_1\right) = Cos^2\alpha$, where the quantities have their usual meanings.
    (b) Find the involutes and evolutes of the circular helix $\vec{r} = \left(a\,Cos\,\theta,\ a\,Sin\,\theta,\ a\,\theta\tan\alpha\right)$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)
Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks. (Calculator Allowed)

1. (a) Derive Newton's forward interpolation formula.
   (b) Estimate population for the year 1905 using Newton's formula for interpolation

Year	1891	1901	1911	1921	1931
Population	98752	132285	168076	195690	246050

2. (a) Describe the two methods of representing any given polynomial in factorial notation.
   (b) Evaluate $\Delta^n \left(\frac{1}{x}\right)$.
3. (a) State and prove Sterling's formula.
   (b) Using Lagrange's interpolation formula find the form of the function $f(x)$ from the table given below.

x	0	1	3	4
y	–12	0	12	24

4. (a) Prove that the *n*th divided difference can be expressed as the quotient of the determinants each of order (n + 1).
   (b) From the given table evaluate $f(7.5)$

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

5. (a) The equation $x^6 - x^4 - x^3 - 1 = 0$ has one root between 1.4 and 1.5. Find the root to four places of decimals by Regula-Falsi method.
   (b) Describe iteration method and use it to find a real root of the equation $f(x) = x^3 + x^2 - 1 = 0$.
6. (a) Explain Newton-Raphson's method geometrically and discuss its failure cases.
   (b) Apply Newton-Raphson's method to find the root of $x^4 - x - 10 = 0$ which is nearer to x = 2 correct to three places of decimals.
7. Find the first and the second derivatives of the function tabulated before at the point 1.1.

x	1	1.2	1.4	1.6	1.8	2.0
f(x)	0	0.1280	0.5440	1.2960	2.4320	4.00

8. (a) Solve $y_{x+2} - 7_{x+1} - 8y_x = (x^2 - x)2^x$.
   (b) Obtain the square root of 11 to five places of decimals by Newton's-Raphson method.
9. Calculate the value of $\int_4^{5.2} \log x \, dx$ by Simpson's ⅓ rule.
10. (a) Find $\int_0^4 \frac{dx}{1 + x^2}$ by using Simpson's ⅜ rule and hence obtain an approximate value of π.
    (b) Show that $y_x = C_1 + C_2\,2^x - x$ is a solution of the difference equation $y_{x+2} - 3y_{x+1} + 2y_x = 1$.

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Examination Programme, 2016

M.Sc. Mathematics, Part-II

Date	Papers	Time	Examination Centre
01.06.2016	Paper-IX	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
03.06.2016	Paper-X	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
07.06.2016	Paper-XI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
09.06.2016	Paper-XII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
11.06.2016	Paper-XIII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
13.06.2016	Paper-XIV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
15.06.2016	Paper-XV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
17.06.2016	Paper-XVI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–X
(Functional Analysis)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that a normed linear space is a metric space under the property $\left| \|x\| - \|y\| \right| \leq \|x\| - \|y\|$.

   (b) If p is a real no such that $1 \leq p < \infty$ and let l_∞^n be the space of all ordered n-tupples of scalars real or complex as $x = (x_1, x_2, ---, x_n)$ with the norm defined by $\|x\| = \max\{|x_1|, |x_2|, ---, |x_n|\}$. Then show that l_∞^n is a Banach space.

2. Prove that the space C [0, 1] of all complex valued function on [0, 1] is not a Banach space with respect to the norm defined by $\|f\| = \int_0^1 |f(t)| \, dt$.

3. If $\|x\|$ and $\|x\|^1$ generate the same topology on a linear space L, then show that these norms are equivalent.

4. Consider the linear space of all bounded sequences $x = (x_1, x_2, x_3, ---, x_n, ---)$ of scalars. Define $\|x\|_\infty = \underset{n}{Sup.} |x_n|$ and denote this space by l_∞. Then show that l_∞ is a Banach space.

5. (a) Show that the inner product space is jointly continuous.

   (b) If x and y are any two vectors in an inner product space then prove that $|(x, y)| \leq \|x\| \cdot \|y\|$.

6. State and prove closed graph theorem.

7. (a) Prove that every Hilbert space is reflexive.

   (b) Prove that an operator T on H(Hilbert space) is normal iff $\|T^*(x)\| = \|T(x)\| \; \forall \; x \in H$.

8. (a) State and prove Riesz lemma.

   (b) If x and y are two vectors on a Hilbert space H then show that
   $$\|x + y\|^2 + \|x - y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right).$$

9. If $\{e_1, e_2, e_3, ---, e_n\}$ is an orthonormal set in a Hilbert space H and if x is an arbitrary element in H then show that $x = \sum_{i=1}^{n} (x, e_i) e_i$.

10. (a) Show that a normed linear space N can be embedded into N**.

    (b) Show that any two normal linear spaces having the same finite dimension are homeomorphic.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XI

(Partial Differential Equations)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Find the general solution of linear partial differential equation
$px(x+y) = qy(x+y) - (x-y)(2x+2y+z)$

(b) Construct the general integral of the equation $(x-y)p + (y-x-z)q = z$ and particular solution through the circle $z = 1,\ x^2 + y^2 = 1$.

2. (a) Solve $p_1 x_1 + p_2 x_2 = p_3^2$ using Jaicobi's method.

(b) Find the complete solution of $2zx - px^2 - 2qxy + pq = 0$ using Charpit's method.

3. (a) Solve the second order partial differential equation $4r - 4s + t = 16 \log(x+2y)$.

(b) Construct the general solution of $xyr + x^2 s - yp = x^3 e^y$.

4. (a) Find the integral surface of the linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$. which contains the straight line $x + y = 0,\ z = 1$.

(b) Construct the orthogonal surface of the given surface $f(x, y, z) = c$.

5. Solve the following partial differential equations.

(a) $\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = Cos\ mx\ Cos\ ny$

(b) $(D - D' - 1)(D - D' - 2)z = Sin(2x + 3y)$

6. Describe the working rule for finding C.F. of reducible non-homogeneous linear partial differential equation with constant co-efficient.

7. Reduce the following P.D.Es to canonical form.

(a) $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$

(b) $r + 2xs + x^2 t = 0$

8. Transform the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ in the cylindrical co-ordinates (r, ϕ, z).

9. (a) Find the general solution of heat equation when both ends of a bar are kept at zero temperature and the initial temperature is given.

(b) Obtain the general solution of the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ given that the initial deflection $u(x, 0) = f(x)$ and the initial velocity $\left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x)$.

10. Find the temperature distribution on inside a square plate of side having boundary conditions $u(o, y\ t) = u(a, y, t) = u(x, o, t) = u(x, a, t) = 0$ and initial condition $u(x, y, o) = Cos\frac{\pi(x-y)}{a} - Cos\frac{\pi(x+y)}{a}$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XII
(Analytical Dynamics)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that in a simple dynamic system T + V = Constant. Where T and V have their usual meaning.
   (b) What are Holonomic and Non-Holonomic dynamical systems, expalin them with suitable examples.
2. (a) In a holonomic dynamical system derive Lagrange's equation of impulsive motion.
   (b) A bead is sliding on a uniformly rotating wire in a force free space. Derive the equation of motion.
3. (a) Explain normal co-ordinates and describe small oscillation under constraint in terms of normal co-ordinates.
   (b) Prove that the roots of the Lagrangian determinant in the theory of small oscillation are real and positive.
4. What do you mean by small oscillation, explain clearly and describe the Lagrange's method of solution in this situation.
5. (a) Explain the principle of least action and hence establish it in terms of arc length of a particle path.
   (b) A particle moves in a plane under a central force depending on its distance from the origin. Then construct the Hamiltonian of the system and derive Hamilton's equation of motion.
6. (a) Define the generating function of a transformation and give an example of a generating function of transformation.
   (b) Show that the transformation $Q = Log\left(\frac{1}{q} Sin\, p\right)$; $P = q\, Cot\, p$ is canonical.
7. State and prove Jaicobi-Poisson theorem.
8. Determine the kinetic energy and the moment of momentum of a rigid body rotating about a fixed axis.
9. Define Poisson's Bracket and show that the Poisson's Bracket obeys the distributive law i.e.
   (a) $[u + v, w]_{qr,\, pr} = [u, w]_{qr,\, pr} + [v, w]_{qr,\, pr}$
   (b) $[uv, w]_{qr,\, pr} = u[v, w]_{qr,\, pr} + [u, w]v_{qr,\, pr}$
10. Derive Euler's equation of motion for the motion of a rigid body about a fixed pont.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIII
(Fluid Mechanics)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Derive the equation of continuity in spherical polar co-ordinates.
   (b) Determine the stream lines and path of particles whose velocity components are $u = \frac{x}{1+t}, \quad v = \frac{y}{1+t}, \quad w = \frac{z}{1+t}$.

2. Obtain Euler's equation of fluid motion.

3. Derive the equation of energy in the motion of no-viscous fluid.

4. (a) Derive pressure equation.
   (b) Prove that if the motion of an ideal fluid for which the density is a function of pressure p only, is steady and the external forces are conservative, then there exists a family of surfaces which contain the stream lines and vortex lines.

5. (a) Distinguish between source and sink. Find the complex potential due to a source of strength m placed at the origin.
   (b) Show that $u = 2Axy, \ v = A\left(a^2 + x^2 - y^2\right)$ are the velocity components of a possible fluid motion. Determine the stream function of fluid motion.

6. (a) Determine boundary surface and find the condition that $f(x, y, z, t) = 0$ may be a boundary surface.
   (b) Show that $u = -\frac{2xyz}{\left(x^2+y^2\right)^2}, \ v = \frac{\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}$ and $w = \frac{y}{x^2+y^2}$ satisfy the irrotational motion.

7. (a) State and prove Kelvin's circulation theorem.
   (b) A velocity field is given by $\vec{q} = \frac{-\hat{2}y + \hat{j}x}{x^2+y^2}$. Determine whether the follow is irrotational ? Also calculate the circulation round a unit circle with centre at the origin.

8. (a) Show that in two dimensional irrotational motion, stream function satisfies Laplace's equation.
   (b) A two dimensional flow field is given by $\Psi = xy$ then
      (i) Show that the flow is irrotational.
      (ii) Find the velocity potential.
      (iii) Find the stream lines and potential lines.

9. Derive Cauchy-Riemann differential equation in polar form.

10. (a) Define stress tensor and strain tensor and develop the relation between stress tensor and rate of strain tensor.
    (b) The stress tensor at a point P is given by $\delta_{ij} = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{bmatrix}$. Determine the stress vector on the plane through P whose unit normal is $\hat{n} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIV
(Operation Research)
Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Define hyper plane and hyper sphere. Prove that every hyper plane in R^n is a convex set.
   (b) Find basic feasible solution of the system $2x_1 + x_2 + 4x_3 = 11,\ 3x_1 + x_2 + 5x_3 = 14$.

2. (a) Reduce feasible solution $x_1 = 2, x_2 = 4\ and\ x_3 = 1$ of the system $2x_1 - x_2 + 2x_3 = 2$ and $x_1 + 4x_2 = 18$ to a basic feasible solution and mention its kind.
   (b) Solve graphically L.P.P.
   Min $z = 5x_1 + 3x_2$
   Subject to the condition : $x_1 + x_2 \le 6,\ 2x_1 + 3x_2 \ge 6,\ 0 \le x_1 \le 4,\ \text{and}\ 0 \le x_2 \le 3$.

3. Using Simplex method solve L.P.P.
   Max $z = 4x_1 + 10x_2$
   Subject to the condition : $2x_1 + x_2 \le 50,\ 2x_1 + 5x_2 \le 10,\ 2x_1 + 3x_3 \le 90;\ x_1 \ge 0, x_2 \ge 0$.

4. Apply two phase simplex method to compute;
   Maximize $z = 5x_1 - 4x_2 + 3x_3$
   Subject to the condition : $2x_1 + x_2 - 6x_3 = 20,\ 6x_1 + 5x_2 + 10x_3 \le 76,\ 8x_1 - 3x_2 + 6x_3 \le 50$
   and $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$.

5. Construct the dual problem of the L.P.P.
   Maximize $z = 3x_1 + x_2 + 2x_3 - x_4$
   Subject to the condition : $2x_1 - x_2 + 3x_3 + x_4 = 1,\ x_1 + x_2 - x_3 + x_4 = 3$
   and $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$ and x_4 are unrestricted.

6. (a) Prove that the dual of the dual of a primal problem is the primal problem itself.
   (b) Describe the dual simplex method by elaborating each step clearly.

7. For the L.P.P. Maximize $z = 3x_1 + 4x_2 + x_3 + 7x_4$
   Subject to the condition :
   $8x_1 + 3x_2 + 4x_3 + x_4 \le 7$
   $2x_1 + 6x_2 + x_3 + 5x_4 \le 3$ and $x_i \ge 0\ (i = 1, 2, 3, 4)$.
   $x_1 + 4x_2 + 5x_3 + 2x_4 \le 8$
   Describe the effect of discrete change in a_{ij} an element of co-efficient matrix

8. (a) Describe the method of, constructing the solution of "Game" problem where the game is without saddle point.
   (b) Solve the game problem whose pay off matrix is $\begin{bmatrix} 6 & 2 & 7 \\ 1 & 9 & 3 \end{bmatrix}$.

9. Explain Fibonacci method of solution of non-linear programming problem.

10. (a) Obtain the feasible solution of the N.L.P.P. :—
    Maximize $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
    Subject to the condition : $x_2 \le 8,\ x_1 + x_2 \le 10\ \text{and}\ x_1, x_2 \ge 0$.
    (b) Use Lagrange's multiplier method to solve the non linear programming problem :—
    $z = ax_1^2 + bx_2^2 + cx_3^2$, where $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 1$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XV

(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Define inner and outer product of two tensors and prove that the outer product of two tensors is a tensor of rank equal to the sum of ranks of the two tensors.
   (b) Show that any linear combination of tensors of the type (r, s) is a tensor of the type (r, s).

2. (a) What do you mean by Chritoffel symbols ? Prove that $[ij, k] + [jk, i] = \frac{\partial}{\partial x^j} gik$.
   (b) Derive the law of transformation of Christoffel symbols of second kind.

3. (a) State and prove convolution theorem on inverse Laplace transform.
   (b) Find the Laplace transform of the functions (i) $Sin\, h^2\, at$, (ii) $\frac{e^{at} - 1}{a}$.

4. Apply Laplace transform to solve $(D^3 - 2D^2 + 5D)y = 0$ if y = 0, $\frac{dy}{dt} = 1$ at t = 0 and y = 1 at $t = \frac{\pi}{8}$.

5. (a) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{1}{s^2(s^2 - a^2)}\right\}$.
   (b) Find the relation between Fourier transform and Laplace transform.

6. (a) Explain about the Firedholm integral equations of three kinds.
   (b) Solve the Firedholm integral equation of second kind by successive substitution.

7. (a) Discuss Firedholm integral equation and Voltera integral equation.
   (b) Verify that the function $u(x) = 1 - x$ is a solution of the integral equation $\int_0^x e^{x-1} u(t)dt = 0$.

8. (a) Form a Voltera integral equation. Corresponding to the differential equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 3y = 0$ with initial conditions $y(0) = 1$ and $y'(0) = 0$.
   (b) Define orthogonality of two functions on an interval [a, b]. If k(x, t) is symmetric, $f_1(x)$ and $f_2(x)$ are eigenfunctions of k(x, t) corresponding to eigenvalues λ_1 and λ_2 $(\lambda_1 \neq \lambda_2)$ respectively. Then show that the functions $f_1(x)$ and $f_2(x)$ are orthogonal on [a, b].

9. The maintenance and re-sale value per year of a machine whose purchase prices is Rs. 7000/- is given below :—

Year	1	2	3	4	5	6	7	8
Maintenance Cost in Rs.	900	1200	1600	2100	2800	3700	4700	5900
Re-sale Value in Rs.	4000	2000	1200	600	50	400	400	400

When should the Machine be replaced.

10. Discuss the deterministic model with instantaneous production (Shortage not allowed).

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XVI
(Programming in 'C')

Annual Examination, 2016

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. What is an operator ? Describe different types of operators that are included in C with examples.
2. What is an expression ? What are its components ?
3. Are the library functions actually the part of the C language ? Explain. How are the library functions accessed ?
4. Explain the difference between while loop and do while loop in C with examples. What is the purpose of switch statement in C. Explain with the help of an example.
5. Write a program to multiply 3×3 matrices in C.
6. What is an Array ? How does an Array differ from an ordinary variable ?
7. What is a function ? State three advantages of using functions. What is the purpose of return statement ?
8. What is the purpose of type def feature ? Explain with an example how this feature is used in structure ?
9. Write a program in C to find Factorial of a given number.
10. What is recursion ? Write a program to find the square roots of a quadratic equation.

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M.Sc. Mathematics, Part–II, Paper–XVI (Practical)
Counselling & Examination Programme, 2016

Practical Counselling Programme

Enrollment No.	***Date***	***Time***	***Venue***
All Old & New Students	21.06.2016 to 27.06.2016	1:00 PM to 5:00 PM	School of Computer Education (IT) Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001

Practical Examination Programme

Enrollment No.	***Date***	***Time***	***Venue***
All Old & New Students	28.06.2016	11.30 AM to 2.30 PM	School of Computer Education (IT) Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XVI [Practical]

(Programming in 'C')

Annual Examination, 2016

Time : 3 Hours. **<u>SET–I</u>** **Full Marks : 20**

Answer any Two Questions. All questions carry equal marks.

1. Write a C Program to generate sum of first 10 even numbers.
2. Write a program in C to solve a quadratic equation.
3. Write a program in C to check whether the input number is prime.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XVI [Practical]

(Programming in 'C')

Annual Examination, 2016

Time : 3 Hours. **<u>SET–II</u>** **Full Marks : 20**

Answer any Two Questions. All questions carry equal marks.

1. Write a C Program to generate sum of first 10 odd numbers.
2. Write a program in C to calculate the factorial of the given number using recursion.
3. Write a program in C to generate Fibonacci series upto given n term.

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