Nalanda Open University

Annual Examination - 2015

B.Sc. Mathematics (Honours), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) Define equivalence relation and equivalence classes of sets. Give at least two examples of equivalence relation (with full justifications).
   (b) Construct an example of a relation which is neither reflexive nor symmetric nor transitive.
2. (a) What do you mean by partial and total order relation?
   (b) Define a lattice and specify condition under which a lattice is said to be complete.

Group - B

3. (a) Give algebraic definition of a lattice. Also justify for a set to be algebraic lattice.
   (b) A relation R is reflexive and circular iff R is reflexive and triangular. Prove this.
4. Let $f : A \to B$ be one to one mapping of A to B. Then show that there exists a unique one to one mapping $g : B \to A$ such that gof=I_A, the identity mapping on A and fog = I_B, the identity mapping on B.
5. Introduce the idea of denumerable set. Prove that any denumerable union of denumerable sets is a denumerable set.

Group - C

6. Exhibit the multiplication of two matrices by stating conformability of product of matrices.
7. (a) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, find the value of $A^2 - 4A + 3I$, where I is the unit matrix with proper dimension.
   (b) By taking three matrices A, B, C compatible for multiplication so the AB, BC, (AB)C and A(BC) are defined, prove that (AB)C = A(BC)
8. Introduce transpose of a matrix. Prove that the transpose of the sum of two matrices is equal to the sum of transposes of the two matrices.

Group - D

9. Find the condition that the equation $x^4-px^3-qx^2+rx+s=0$ may have its roots in arithmetical progression and solve the equation $x^4-2x^3-21x^2+22x+40=0$.
10. (a) Describe the Cardon's method to solve the cubic equation $a_0x^3+3a_1x^2+3a_2x+a_3=0$.
    (b) Use Euler's method to solve the biquadratic equation $x^4-3x^2-6x-2=0$.

Group - E

11. (a) Find the expansion of Sin $\propto$ in terms of ascending powers of $\propto$.
    (b) Prove that log i=i $(2n+\frac{1}{2})\pi$, where $n \in Z$.
12. (a) State and establish Gregories series for the expansion of $\tan^{-1}x$ in the ascending power of x
    (b) If $\sin^{-1}(x+iy)= \propto+i\beta$. Prove that (i) $\frac{x^2}{Cos\alpha^2\beta}+\frac{y^2}{Sin\alpha^2\beta}=1$ (ii) $\frac{x^2}{Sin^2\propto}-\frac{y^2}{Cos^2\propto}=1$

Nalanda Open University
Annual Examination - 2015
B.Sc. Mathematics (Honours), Part-I
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. State and prove Leibnitz's theorem.
2. (a) If $y = \text{Sin}\,(m\,\text{Sin}^{-1}x)$, show that :
   (i) $(1 - x^2)y_2 - xy_1 + m^2y = 0$ (ii) $(1 - x^2)y_{n+2} - (2n + 1)y_{n+1} + (m^2 - n^2)y_n = 0$
   (b) If $y = (x^2 - 1)^n$, then show that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n\,(n + 1)y_n = 0$
3. (a) Use Tayler's theorem in Lagrance form of remainder for the expansion of $\text{Sin}^{-1}(x + h)$.
   (b) Find the Lagrange's form of remainder after n terms in the expansion of $e^{ax}\,\text{Cos}bx$ in powers of x.

Group - B

4. (a) Find the radius of curvature in pedal form.
   (b) Derive the equation of the tangent line to the curve $y = be^{-x/a}$ at the point where curve crosses the y-axis.
5. Evaluate *any Two* of the following integrals :
   (i) $\int \frac{dx}{x(x^2+1)^3}$ (ii) $\int \frac{1+x^{1/2}}{1+x^{1/3}}dx$ (iii) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$, $(\alpha < x < \beta)$
6. (a) Find the limit, when n tends to infinity of the series $\sum_{n=1}^{n-1}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}$.
   (b) Evaluate $\underset{n\to\infty}{Lim}\left(\frac{\lfloor n}{n}\right)^{1/n}$
7. Trace the curve whose parametric equations are $x = a\,(\theta + \text{Sin}\theta)$, $y = a\,(1 + \text{Cos}\theta)$. Hence, find the whole area of the curve.

Group – C

8. (a) Find the pole of $3x + 14y + 1 = 0$ with respect to the conic $x^2 + 5xy + 4y^2 - 8x + 5 = 0$.
   (b) Find the polar equation of the normal at any point $\rho(\propto)$ of the Conic $\frac{l}{r} = 1 - eCos\theta$.
9. Find the centre of the Conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

Group – D

10. (a) Find the equations of the spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y - 15 = 0$.
    (b) Find the equation of the right circular cone whose axis is the x-axis, vertex is the origin and the semi-vertical angle is $\frac{\pi}{3}$.
11. Find the condition that the general equation of second degree given by $ax^2 + 2hxy + by^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$, represents a cone.

Examination Programme, 2015

B.Sc (Part – I) All Honours Subjects

(Except Home Science and Geography Honours)

Date	Papers.	Time	Examination Centre
23/3/2015	(Hons) P-I	**3.30 to 6.30 pm**	Nalanda Open University, Patna
25/3/2015	(Hons) P-II	**3.30 to 6.30 pm**	Nalanda Open University, Patna
28/3/2015	Rastrabhsha-100 orHindi +Urdu 100-I	**3.30 to 6.30 pm**	Nalanda Open University, Patna
30/3/2015	Botany (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
31/3/2015	Math (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
01/4/2015	Geography (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
02/4/2015	Chemistry (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
03/4/2015	Home Scince (Sub)-P I	8 to 11 am	Nalanda Open University, Patna
04/4/2015	Zoology (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
06/4/2015	Physics (Sub) P-I	8 to 11 am	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2015

B.Sc. Mathematics (Subsidiary), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting atleast **One** question from each group.*
All questions carry equals marks.

Group - A

1. (a) What do you mean by a relation on a non-empty set A? When a relation on a set will be called reflexive and transitive. Give examples of each of these relations.
   (b) Prove that in the set of integers the relation "a divides b" is reflexive and transitive.
2. If R_1 and R_2 are two equivalence relations on a set A, then show that $R_1 \cap R_2$ is also can equivalence relation.
3. (a) Define a bijective map of a non-empty set A to a non-empty set B. Give two examples of bijective maps.
   (b) Let X be all points in a plane and r a relation on X such that "arb" iff a and b are equi-distant from the origin. Describe the equivalence classes.
4. Define an abelian group. Show the set of integers under addition is an abelian group, while the same set is a non-abelian group under multiplication.

Group - B

5. (a) Use DeMoivre's theorem to extract the roots of the equation $x^7 + 1 = 0$.
   (b) Find the expansion of $\sin\alpha$ in terms of ascending powers of α.
6. (a) Prove that $\tan\left(i\log\frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2-b^2}$, where a and b are two real numbers.
   (b) Decompose $\log(x + iy)$ into real and imaginary parts.

Group - C

7. (a) Introduce the idea of a monotonic sequence and prove that a monotonically decreasing sequence converges to its greatest lower bound.
   (b) Show that the sequence (x_n), where $x_n = \left(1+\frac{1}{n}\right)^n$ is a convergent sequence.
8. (a) What do you mean by radical axis of two circles? Determine the radical axis of the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$.
   (b) Find the conditions under which a general equation of second degree represents an ellipse.

Group - D

9. (a) If $y = \sin(m\sin^{-1}x)$, show that $(1 - x^2)y_2 - xy_1 + m^2y = 0$
   (b) If under certain circumstances the function $f(x + h)$ can be expanded in the powers of h, then the expansion can be $f(x+h) = f(x) + \frac{h}{\lfloor 1}f'(x) + \frac{h^2}{\lfloor 2}f''(x) + ... + \frac{h^n}{\lfloor n}f^n(x) + ...\infty$. Prove this statement.
10. (a) State and prove Euler's theorem on homogeneous function of two variables.
    (b) Evaluate the limits : $\lim_{x\to 0} x^{\sin x}$ (ii) $\lim_{x\to\frac{\pi}{2}} (\tan x)^{\cos x}$.
11. (a) Give the geometric meaning of scalar triple product.
    (b) Prove that $\vec{r} = (\vec{r}.\hat{i})\hat{i} + (\vec{r}.\hat{j})\hat{j} + (\vec{r}.\hat{k})\hat{k}$.

Nalanda Open University
Annual Examination - 2015
B.Sc. Mathematics (Honours), Part-II
Paper-III

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) Define sum of two cuts and prove that the sum of two cuts is also a cut.
   (b) Show that any non-empty open set is union of open intervals.
2. (a) Define a closed set. Prove that the intersection of any number of closed sets, is closed.
   (b) Introduce the concept of a compact set and give an example of it.
3. (a) What do you understand by totally discontinuous function.
   (b) Give the meaning of sign of derivative of a function.

Group-B

4. (a) State and prove Taylor's theorem.
   (b) State Lagrange's mean value theorem and interpret it geometrically.
5. (a) Give the idea of improper integral supported by suitable example.
   (b) By defining Beta function and Gamma functions, prove the relation between them is given by $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
6. (a) Prove that every bounded monotonically increasing sequence converges to its least upper bound.
   (b) Let $x_1=1, x_2=\sqrt{2+x_1}, x_3=\sqrt{2+x_2}, ..., x_{n+1}=\sqrt{2+x_n}, ...$ Show that the sequence (x_n) is convergent and then the limit of convergence is 2.

Group-C

7. (a) If $\sum u_n$ and $\sum v_n$ are two infinite series of positive terms such that $\lim\limits_{n\to\infty} u_n/v_n = l(>0)$, then the two series converge or diverge together.
   (b) Test the convergence of the series
   $$x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+.............+\frac{n^2-1}{n^2+1}x^n.............to\infty.$$
8. (a) Introduce the idea of Absolutely and conditionally convergent series with the support of suitable examples.
   (b) Rearrange terms of series $\sum_1^\infty \frac{(-1)^{n-1}}{n}$ so that it converges to $\frac{1}{2}\log 6$.
9. (a) Define a vector space over a field and prove that the set C of all complex numbers, under the addition of complex numbers together with real number multiplication of complex numbers forms a vector space over the field of real numbers.
10. (a) Define eigen values and eigen vectors of a square matrix. Compute the eigen values and eigen vectors of the matrix $\begin{bmatrix}2 & 1 & 0\\ 0 & 2 & 1\\ 0 & 0 & 2\end{bmatrix}$
    (b) Determine the rank of the matrix $\begin{bmatrix}1 & 2 & -1\\ 3 & -1 & 2\\ 2 & -2 & 3\\ 1 & -1 & 1\end{bmatrix}$

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Nalanda Open University
Annual Examination - 2015
B.Sc. Mathematics (Honours), Part-II
Paper-IV

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) Solve any two of the following differential equations

(i) $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6 = 0$ (ii) $p^2 - p(e^x + e^{-x}) + 1 = 0$

(iii) $xyp^2 - (x^2 - y^2)p - xy = 0$

(b) Find the orthogonal trajectory of the family of cardoids r=a(1+Cos θ)

2. Describe the method of construction of Auxiliary equation in the case of second order linear differential equation say $\frac{d^2y}{dx^2} + P_1\frac{dy}{dx} + P_2 y = 0$

3. (a) Apply the method of variation of parameters to construct the general solution of the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

(b) Solve by the method of variation of parameters, to the differential equation $\frac{d^2y}{dx^2} + ny^2 = \sec nx$

Group-B

4. (a) Prove that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$ and check that $\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$

(b) Prove that $\vec{r} = (\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k}$

5. If $\vec{a}$ *is an* unit vector, then show that $\left|\vec{a}\times\frac{d\vec{a}}{dt}\right| = \left|\frac{d\vec{a}}{dt}\right|$

6. (a) Give the geometric interpretation of gradient of a scalar function.

(b) If $\vec{a}$ *is a* constant vector, prove that (i) $\nabla(\vec{a}\cdot\vec{r}) = \vec{a}$ (ii) $\nabla\cdot(\vec{a}\cdot\vec{r}) = 0$ (iii) $\nabla\times(\vec{a}\cdot\vec{r}) = 2\vec{a}$

Group-C

7. State and prove Gauss theorem.

8. (a) Derive the necessary and sufficient condition for the equilibrium of co-planar forces acting on a body.

(b) Three forces, each equal to P, act along the sides of the triangle ABC taken in order. Prove that the magnitude of the resultant is $P\sqrt{1-8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$. Find the equation of the line of action of the resultant force and its distance from A.

Group-D

9. (a) Explain virtual displacement and virtual work. What are the forces which are omitted in forming the equation of the virtual work of the system.

(b) A uniform beam of length & rests with its end on two smooth planes, which intersect in a horizontal line. If the inclination of the planes to the horizontal are α and β ($\beta > \alpha$). Show that the inclination θ of the beam to horizontal in one of the equilibrium positions is given by $\tan\theta = \frac{1}{2}(Cot\alpha - Cot\beta)$.

10. (a) Derive the tangential and normal velocities in polar co-ordinates.

(b) A particle of mass m moves in a straight line under a force mn^2x(distance) towards a fixed point in the straight line and under small resistance to its motion equal to mμ(velocity). Discuss the motion.

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Nalanda Open University
Annual Examination - 2015
B.Sc. Mathematics (Subsidiary), Part-II
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Eight questions, selecting atleast one from each group. All questions carry equal marks.

Group-A

1. (a) Evaluate any two of the following integrals :

(i) $\int \frac{x^2 - x + 1}{(x+1)\sqrt{x-2}} dx$ (ii) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (iii) $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$ $[\alpha, \beta]$

2. (a) Evaluate $\underset{n\to\alpha}{Lt} \frac{[(n+1)(n+2)(n+3)..........(n+n)]^{y_n}}{n}$.

(b) Find the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x dx$

3. Compute the following definite integrals :

(i) $\int_0^{\pi/2} \frac{\sin 2x dx}{\sin^4 x + \cos^4 x}$ (ii) $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)$ (iii) $\int_0^{\alpha} \frac{\log(1+x^2)}{1+x^2} dx$

4. (a) Find the entire length of the Cardoid $r = a(1 + \text{Cos}\theta)$.

(b) Find the area enclosed by the curve $xy^2 = a^2(a - x)$.

5. Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the (i) major axis (ii) minor axis.

6. Solve any two of the following differential equations :

(i) $\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = x^2 + xy$ (ii) $p^2 - p(e^x + e^{-x}) + 1 = 0$ (iii) $y = 2px + p^2$

7. Find the general solution of any two of the following equations :

(i) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$ (ii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4y = x^2$ (iii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$

Group – B

8. (a) Find the equation of the sphere which passes through the point (α, β, γ) and the circle $x^2 + y^2 + z^2 = a^2, z = 0$.

(b) Prove that the equation of a cone passing through the Co-ordinate axes, if $fyz + gzx + hxy = 0$.

9. Find the equation of the right circular cylinder, whose axis is given by $\frac{x}{1} = \frac{y}{0} = \frac{z}{2}$ and radius $\sqrt{7}$.

Group – C

10. (a) Define linear dependence and independence of vectors. Show that the vectors (1,0,0), (0,1,0) and (0,0,1) are linear independent.

(b) What is hyper plane and convex set? Prove that a hyperplane is a convex set.

11. Define convex combination of vectors $v_1, v_2, v_3, \ldots\ldots, v_m$. Any point on the line segment joining two points in R^n cab be expressed as a convex combination of two points. Prove this statement.

[See Back Page]

Group – D

12. Deduce general conditions for equilibrium of a system of forces.

13. A heavy uniform rod AB is a hinged at A and a force F is applied at the lower end B. If the rod makes an angle 60° with the horizontal in the position of equilibrium, then find the reaction at the hinge and magnitude of force F.

Group – E

14. (a) Explain the principle of conservation of linear momentum.

    (b) Show that the moment of force or torque about the origin O of a co-ordinate system, is equal to the time rate of change of angular momentum.

15. (a) State and establish the principle of energy.

    (b) A shot of mass *m* is projected from a gun of mass M by an explosion with greates K.E. (E) show that the gun recoils with a velocity $\sqrt{\dfrac{2ME}{M(M+m)}}$.

16. (a) For a simple harmonic motion (S.H.M), derive the expression for periodic time after defining it.

    (b) Analyze the motion of a body under inverse square law.

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Nalanda Open University

Annual Examination - 2015

B.Sc. Mathematics (Honours), Part-III

Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Let x be a metric space, then show that for each pair of distinct points of x, there exist neighbourhoods N_1 and N_2 such that $N_1 \cap N_2 = \phi$.
   (b) Let (X, d) be a metric space and ρ be defined on X × X as $\rho(x, y) = \frac{d(x,y)}{1+d(x,y)}$, then show that ρ is a metric on X.
2. (a) Let (X, d) be a metric space and $E \subseteq X$, then show that E is closed iff E contains each of its accumulation points.
   (b) In a metric space (X, d), prove that each closed sphere is a closed set.
3. (a) In a metric space, show that the limit of a convergent sequence is unique.
   (b) If x only are two points in the metric space (X, d) and if (X_n) (Y_n) are convergent sequences in X having limit of convergence x and y respectively, then show that $d(x_n, y_n)$ converges to $d(x, y)$.
4. State and prove Cantor's intersection theorem.
5. Define a complete metric space. The metric space (C, d) of the complex plane, where $d(z_1, z_2) = |z_1 - z_2|$ for all z_1, z_2 is complete.

Group 'B'

6. Define a topological space. Let (X, d) be a metric space space and T be the family of those sub-sets E_i of X, which are open with respect to metrical. Then prove that T is a topology on X.
7. What is a Hausdorff space? Let (X, T) be a topological space. Then (X, T) is a Hausdorff space iff for each pair of distinct points x, y of X, there exist a pair of disjoint open set E and F such that $x \in E$ and $y \in F$. Prove this statement

Group 'C'

8. State and prove Darboux's theorem.
9. Introduce the idea of an oscillatory sum in Riemann integration. Prove that a necessary and sufficient condition for R-integrability of bounded function f over an interval $[a, b]$ is that for every $\in > 0$, there exists a partition ρ such that the oscillatory sum is less than $\in$.

Group 'D'

10. (a) Show that every bounded monotonic function over a closed interval $[a, b]$ is Riemann Integrable.
    (b) If f is continuous and non-negative on $[a, b]$ and $\int_a^b f(x)dx = 0,$ then prove that $f(x) = 0$, for all $x \in [a, b]$.
11. (a) Discuss the convergence of the series :
    $$1+\frac{1}{3^p}-\frac{1}{2^p}+\frac{1}{5^p}+\frac{1}{7^p}-\frac{1}{4^p}+\frac{1}{9^p}+\frac{1}{11^p}-\frac{1}{6^p}+.....$$
    (b) A normed linear space X is a Banach space iff every absolutely summable series is summable.

For Examination Programme See Back Page

Nalanda Open University

Annual Examination - 2015

B.Sc. Mathematics (Honours), Part-III

Paper-VI

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Define an automorphism. Let G be a group and $x \in$ G, then show that the function f_g defined by :

$f(g) = x^{-1}gx$, $g \in$ G, is an automorphism of G.

(b) Let G be a group. Then for any element g in G, prove that C_G (g) is a sub-group of G.

2. (a) Define an isomorphism of a ring R to a ring T. Prove that for any R, the identity map is an isomorphism of R to R.

(b) What is an ideal (or two sided) of a ring R?

3. (a) Let R be commutative ring with at least two element and with no zero divisions. Then show that the relation ~ defined by $(a, b) \sim (c, d)$ if $ad = bc$, is an equivalence relation on the set of quotients of R.

(b) Define principal ideal ring R and prove that the ring of integers is a principal ideal ring.

Group 'B'

4. (a) What is a prime field? Prove that Q the set of rational numbers is a prime field.

(b) Let R[x] be the set of all polynomials, where a_o, a_y, a_z ... $a_m \in$ R and $a_m \neq o$ as well m is a nor-negative integer. If R is a commutative ring with unity, then show that R[x] is also a commutative ring with unity.

5. (a) Prove that the set of all polynomilas in Z[x] with constant term 0, is a prime ideal in Z[x].

(b) Give examples of two polynomials $f(x)$ and $g(x)$ such that deg (fg) < deg(f) + deg (g). [$d(f)$ means degree of f].

Group 'C'

6. State and prove Caontor's theorem.

7. (a) If E is any set. Then show that Card. ρ(E) = 2, where ρ(E) denotes the power set of E.

(b) For any Cardinal numbers a, b and y prove that :

(i) $\alpha^{\beta} \cdot \alpha^{\gamma}$ (ii) $(\alpha \cdot \beta)^{\gamma} = \alpha^{\gamma} \cdot \beta^{\gamma}$ (iii) $(\alpha^{\gamma})^{0\gamma} = \alpha^{\beta\gamma}$

Group 'D'

8. (a) Define Maximal and Minimal elements in a partially ordered set. Let the set A = {3, 4, 5, 8, 9} be ordered by x divides y. Determine minimal and maximal elements. Justify your answer logically.

(b) By introducing two order types, construct the product of two order types.

9. (a) Find the number of ways in which $(m + n)$ things all distinct, can be divided into two groups of m and n things.

(b) The sum of mean and variance of a binomial distribution is 15 and sum of their squares is 117. Determine the distribution.

10. (a) What are necessary conditions for the differentiability of a complex valued function.

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although the Cauchy-Riemann differential equations are satisfied.

Nalanda Open University

Annual Examination - 2015

B.Sc. Mathematics (Honours), Part-III

Paper-VII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Define a feasible solution of linear programming problem. Prove that the set of all feasible solution of a L.P.P form a convex set.
   (b) Use graphical method to solve the following L.P.P :
   Max $z = 4x_1 + 7x_2$
   Subject to $x_1 + 2x_2 \leq 20$, $x_1 + x_2 \leq 15$, $x_2 \leq 8$, where $x_1 \geq 0$, $x_2 \geq 0$
2. (a) Define convex combination of vectors in R^n. Show that the set of convex combinations of a finite number of linearly independent vectors $v_1, v_2, \ldots\ldots, v_n$ is a convex set.
   (b) A tailor has 95 square meters Cotton and 145 square meters of Wool and dress requires 2 square meters of each. How many of each garment should the tailor produces so as to maximize his income if a suit sells for ₹350 and a dress for ₹145.
3. Use simplex method to solve :
   Maximize $z = 3x_1 + 9x_2$
   Subject to $x_1 + 4x_2 \leq 8$, $x_1 + 2x_2 \leq 4$ and $x_1 \geq 0$, $x_2 \geq 0$.

Group 'B'

4. (a) Solve $(y^2 + yz + z^2)\, dx + (z^2 + zx + x^2)dy + (x^2 + xy + y^2)dz = 0$.
   (b) Solve $\dfrac{dx}{x^2 - yz} = \dfrac{dy}{y^2 - zx} = \dfrac{dz}{z^2 - xy}$.
5. Test for integrability and hence solve the equation $(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$.
6. (a) Give the geometrical interpretation of the equation $Pdx + Qdy + Rdz = 0$.
   (b) Solve the simultaneous equations :
   $$\frac{dx}{dt} + 2x - 3y = t,\ \frac{dy}{dt} - 3x + 2y = e^{2t}$$
7. (a) Use Monge's method to find the complete solution of the equation $2x^2r - 5xys + 2y^2t + 2(px + qy) = 0$.
   (b) Find the orthogonal projection on xz plane of the curves which lie on the paraboloid $3z = x^2 + y^2$ and satisfy the equation $2dz = (x + z)dx + ydy$.

Group 'C'

8. (a) Find the attraction of a uniform solid sphere at an external point.
   (b) A frustum of a uniform thin hollow cone attracts a particle placed at the vertex. Determine the attraction.
9. (a) State and establish Laplace theorem in Cartesian.
   (b) Prove that the half of the potential of a uniform spherical sheet at an external point 0, is due to that portion which is nearer to 0.

Group 'D'

10. (a) Show that the pressure at a point of a fluid at rest, is the same in all directions.
    (b) Three liquids whose densities are A.P fill a semicircular tube whose diameter is horizontal. Show that the depth of one of the common surfaces is the double that of other.
11. (a) Find the depth of the centres of pressure of a triangle immersed in a liquid with the vertex in the surface and base horizontal.
    (b) A body is entire immersed in a liquid being supported by a string. Find the tension of the string.

Nalanda Open University

Annual Examination - 2015

B.Sc. Mathematics (Honours), Part-III

Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Five questions. All questions carry equal marks.

1. (a) (i) Explain the meaning of the operators E and Δ.
   (ii) Show that E and Δ are Commutative with respect to variables.
   (b) Evaluate : (i) $\Delta^3 (1-x)(1-2x)(1-3x)$ and
   (ii) $\Delta^n (e^{ax+b})$ where a and b are constants.
2. (a) Prove that the nth difference of polynomial of nth degree, is constant; when the values of the independent variable are at equal intervals.
   (b) Introduce factorial function and prove that :
   (i) $\Delta^m x^m = h^m \underline{\lfloor m}$ (ii) $Dx^{(-n)} = (x+h)^{(-n)} - x^{(-n)}$.
3. (a) Construct the working table for Backward formula.
   (b) Define divided differences and construct its table.
4. (a) By means of divided differences formula, find the value of $f(2)$, $f(8)$ and $f(15)$ whose table is as under :

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

   (b) Evaluate $\left[\frac{\Delta^2}{E}\right] e^x \; \frac{Ee^x}{\Delta^2 e^x}$.
5. Describe Newton-Gregory formula for Backward Interpolation.
6. Use the Newton-Gregories backward formula for obtaining the derivatives i.e $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots\ldots, \frac{d^ny}{dx^n}$ of interpolated polynomial.
7. (a) Derive Trapezoidal and Simpson's one third rule for numerical integration.
   (b) Solve the difference equation $U_{x+1} = 2^x U_x$.
8. (a) Describe Picard's method of successive approximation.
   (b) Apply Runge-Kutta method for the solution of first order differential equation.
9. (a) State and prove Adam's Predictor formula.
   (b) Describe Milne Corrector formula.
10. (a) Explain Gauss method of elimination for the solution of a system of m equations in m variables.
   (b) Solve the system the system of equations :

$$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = 1$$
$$\frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 = 0$$
$$\frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 = 0$$