NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-I

PAPER–I (Honours)

(Set Theory, Matrices, Abstract Algebra, Theory of Equations and Trigonometry)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Prove that an infinite union of denumerable sets is denumerable.
   (b) Define a Lattice, Complete Lattice and set an example of Lattice which is not a complete Lattice.
2. (a) If $f : x \rightarrow y$ and $A \subseteq y$, $B \subseteq y$ then show that
   $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ *and* $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
   (b) Define an equivalence relation and equivalence classes of sets giving one example of each.
3. State and prove fundamental theorem of equivalence relation.
4. What do you mean by a partial order relation and total order relation and well ordered set. Give one example of each.
5. If A, B, C, D are any three non-empty sets then prove that
   (a) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D) \cup (A \times D) \cup (B \times C)$
   (b) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

GROUP 'B'

6. (a) Prove that if a group G has four elements then it must be abelian.
   (b) Prove that the order of every element of a finite group is a divisor of the order of the group.
7. (a) Define a group and show that the four fourth roots namely 1, -1, i, -i form a group with respect to multiplication.
   (b) Prove that G = {0, 1, 2, 3, 4, 5} is a finite abelian group of order 6 with respect to addition modulo 6.
8. (a) Prove that a group G is abelian if $b^{-1}a^{-1}ba = e \ \forall \ a, b \in G$ and e is the identify element of G.
   (b) If H_1, H_2 are subgroups of a group G then show that $H_1 \cap H_2$ is also a subgroup of G.

GROUP 'C'

9. (a) If A and B are any two non-singular matrices of the same order then prove that $(AB)^{-1} = B^{-1}A^{-1}$.
   (b) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$.
10. Find the eigen values and eigen vectors of the matrix : $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
11. (a) State and prove De-Moiver's theorem.
    (b) Find the condition so that the equation $x^4 - px^3 - qx^2 + rx + s = 0$ may have its roots in arithmetical progression.
12. (a) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Then find the value of $A^2 - 4A + 3I$.
    (b) Solve the following system of linear equations by matrix method.
    $$\left.\begin{aligned} x + y + z &= 6 \\ 2x + y - 3z &= -5 \\ 3x - 2y + z &= 2 \end{aligned}\right\}.$$

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NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-I
PAPER–II (Honours)
(Differential Calculus, Integral Calculus and Analytical Geometry of Three Dimensions)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) State and prove Taylor's theorem.
   (b) Find the Lagrange's form of remainder after n terms in the expansion of $e^{ax}\cos bx$ in powers of x.
2. (a) If $y = e^{a\,Sin^{-1}x}$ then prove that, $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.
   (b) If $y = (x^2-1)^n$ then prove that, $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.
3. (a) Prove that the radius of curvature for the pedal curve $p = f(r)$ is given by $\rho = r\dfrac{dr}{dp}$.
   (b) Find the asymptotes to the curve $(x^2+y^2)(x+2y+2) = x+9y+2$.
4. Evaluate :— (a) $\underset{x\to 0}{Limit}\ \dfrac{xe^x - \log(1+x)}{x^2}$ (b) $\underset{x\to 0}{Limit}\left(\dfrac{\sin x}{x}\right)^{\frac{1}{x^2}}$.
5. (a) If $u = \log(x^2+y^2+z^2-3xyz)$ then show that : $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2} = \dfrac{3}{(x^2+y^2+z^2)^2}$.
   (b) If the normal at any point to the curve $x^{2/3}+y^{2/3} = a^{2/3}$ makes an angle ϕ with x-axis then show that its equation is $y\,Cos\phi - x\,Sin\phi = a\,Cos2\phi$.

GROUP 'B'

6. Evaluate any ***Two*** of the following :—
   (a) $\displaystyle\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$ (b) $\displaystyle\int \frac{x^2}{x^4+1}dx$ (c) $\displaystyle\int \operatorname{cosec}^3 x\ dx$
7. Evaluate any ***Two*** of the following :—
   (a) $\displaystyle\int_0^{\pi/2} \frac{Sin^s x}{Sinx + Cosx}dx$ (b) $\displaystyle\int_0^{\pi/2}\log(Sinx)dx$ (c) $\displaystyle\int_0^{\pi/4}\log(1+\tan\theta)d\theta$
8. (a) Evaluate $\underset{n\to\infty}{Limit}\left(\displaystyle\sum_{n=1}^{n-1}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}\right)$
   (b) Obtain the reduction formula for $\int Cos^m x\, Sin\, nx\, dx$.
9. Find the area of the loop $y^2 = x(x-1)^2$.
10. Find the volume of the solid formed by the revolution of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2} = 1$ about x-axis.

GROUP 'C'

11. (a) If the tangent to the sphere $x^2+y^2+z^2 = r^2$ makes intercepts on the co-ordinate axis a, b, c, respectively then show that $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2} = \dfrac{1}{r^2}$.
    (b) Find the equation of the sphere through the circle $x^2+y^2+z^2 = 5$, $x+2y+3z = 3$ and touch the plane $4x+3y-15 = 0$.
12. (a) Find the polar equation of the conic in the form $\dfrac{\ell}{r} = 1 + e\,Cos\theta$.
    (b) Find the polar equation of the tangent at any point of it to the conic $\dfrac{\ell}{r} = 1 + e\,Cos\theta$.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-I, PAPER–I (Subsidiary)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Eight** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. Let $f: X \to Y,\ A \subseteq Y,\ B \subseteq Y$ then show that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
2. If A, B, C are any three non-empty sets then prove that
   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
3. What do you mean by an Equivalence relation. Give two examples of it.

GROUP 'B'

4. What do you mean by an abelian group ? If a group G has four elements then prove that it must be abelian group.
5. For a finite group G, prove that the order of every element of G is finite and less than or equal to the order of the group G.
6. Let G = {1, w, w^2} where w is an imaginary cube root of unity then prove that G is a group with respect to multiplication as operation.
7. Let f be a homomorphism of a group G onto a group G' with Kernel, K = {x $\in$ G : f(x) = e') where e' is the identity element of G', then show that K is a normal sub group of G.
8. Let f be a homomorphism of a group G into a group G' then prove that.
   (i) f(e) = e' where e is the identity of G and e' is the identify element of G'.
   (ii) $f(a^{-1}) = \{f(a)\}^{-1}\ \forall\ a \in G$.
   (iii) If the order of a $\in$ G is finite then the order of f(a) is the divisor of the order of a.

GROUP 'C'

9. State and prove De-Moivre's theorem.
10. Decompose $\log(\alpha + i\beta)$ into real and imaginary parts.
11. If $\tan(x + iy) = u + iv$ then prove that $u^2 + v^2 + 2u\,Cot\,2x = 1$.

GROUP 'D'

12. Test the convergence of the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots\ldots\ldots\ldots\ \infty$.
13. Test the convergence of the series whose n[th] term is $\left(\sqrt{n^2+1} - \sqrt{n^2-1}\right)$.
14. (a) State and prove Cauchy general principle of convergence of a real sequence.
    (b) Show that the sequence (a_n) where $a_n = \sqrt{n^2 + 4n} - n$ is convergent.

GROUP 'E'

15. Deduce the polar equation of the conic in the form $\frac{\ell}{r} = 1 + e\cos\theta$.
16. (a) State and prove Euler's theorem on Homogeneous functions of two variables.
    (b) If $f(x, y) = x\,Cosy + y\,Cosx$ then prove that : $\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial^2 f}{\partial y\,\partial x}$.
17. Prove that : $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$.
18. Prove that : $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]^2$

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NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-II
PAPER–III (Honours)
Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Define a closed set. Prove that the intersection of any number of closed sets is closed.
   (b) Prove that between two distinct real numbers there lie infinity of irrationals and rationals.
2. (a) State and prove theorem of least upper bound.
   (b) State and prove fundamental theorem of classical analysis.
3. (a) Show that any non-empty open set is a union of open intervals.
   (b) State and prove theorem of greatest lower bound.

GROUP 'B'

4. (a) State and prove Raabe's test.
   (b) Test for the convergence of the series $\Sigma \frac{(n+1)(n+2)}{(n+3)(n+4)}$.
5. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{1+n^2}, \forall\, x > 0$.
   (b) Test the convergence of the series whose n^{th} term is $\sqrt{n^2+1} - \sqrt{n^2-1}$.
6. (a) Test the convergence of the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots\ldots\ldots\ldots \infty$.
   (b) State and prove Cauchy's n^{th} root test for convergence of an infinite series.
7. (a) Define a convergent sequence and show that it is bounded.
   (b) Show that a bounded monotonic increasing sequence tends to its least upper bound.
8. (a) Show that the sequence (a_n) defined by $a_1 = \sqrt{7}$, $a_{n+1} = \sqrt{7 + a_n}$ converges to a positive roof of the equation $x^2 - x - 7 = 0$.
   (b) Let $x_1 = 1, x_2 = \sqrt{2 + x_1}, x_3 = \sqrt{2 + x_2}, \ldots\ldots., x_{n+1} = \sqrt{2 + x_n}$. Show that the sequence (x_n) is convergent and the limit of convergence is 2.

GROUP 'C'

9. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$.
   (b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$.
10. (a) Let V be a vector space and W_1, W_2 are finite dimensional subspaces of V. Then show that $W_1 + W_2$ is finite dimensional and
    dim . W_1 + dim . W_2 = dim. $W_1 \cap W_2$) + dim ($W_1 + W_2$).
    (b) Prove that any two bases of a finite dimensional vector space have the same number of elements.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-II

PAPER–IV (Honours)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Find the orthogonal Trajectory of the family of Cardoids $r = a(1 + Cos\,\theta)$.

   (b) Prove that the system of confocal conic $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal.

2. Solve any **Two** of the following differential equations :—

   (a) $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6 = 0$

   (b) $(px - y)(x - py) = 2p$

   (c) $(x - a)p^2 + (x - y)p - y = 0$

3. (a) Solve $\frac{d^2y}{dx^2} + a^2y = Sec\,ax$ by using variation of parameters.

   (b) Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$ by using method of change of variables.

GROUP 'B'

4. (a) Show that $\left[\vec{a}\times\vec{b}\;\;\vec{b}\times\vec{c}\;\;\vec{c}\times\vec{a}\right] = \left[\vec{a}\;\vec{b}\;\vec{c}\right]^2$.

   (b) Prove that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c}$.

5. (a) Prove that $\frac{d}{dt}(\vec{u}\times\vec{v}) = \vec{u}\times\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\times\vec{v}$.

   (b) Prove that $\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \vec{u}\cdot\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\cdot\vec{v}$.

6. (a) Prove that $\nabla\times(\vec{u}\pm\vec{v}) = \nabla\times\vec{u}\pm\nabla\times\vec{v}$.

   (b) Prove that : $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot(\nabla\times\vec{u}) - \vec{u}\cdot(\nabla\times\vec{v})$

7. State and prove the necessary and sufficient condition of the principle of virtual work.

GROUP 'C'

8. Derive the tangential and normal velocities and accelerations in polar co-ordinates.

9. State and prove the necessary and sufficient condition for equilibrium of a system of co-planar forces also find the equation of line of action of the resultant.

10. Define simple Harmonic motion. If in a simple harmonic motion u, v, w be the velocities at distances a, b, c from a fixed point on the straight line which is not the centre of the force. Show that the periodic time T is given by the equation:

$$4\pi^2(a-b)(b-c)(c-a) = T\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$

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NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-II
PAPER–II (Subsidiary)
Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Eight** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP-A

1. Evaluate any two of the following integrals :–

 (a) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ (b) $\int \frac{dx}{\sin x(3+2\cos x)}$ (c) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

2. Evaluate any two of the following :–

 (a) $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx$ (b) $\int_0^{\pi/4} \log(\tan x) dx$ (c) $\int_0^{\pi/2} \frac{Sin^2 x}{Sinx + Cosx} dx$

3. Find the reduction formula for :–

 (a) $\int_0^{\pi/2} Sin^m x \ Cos^n x \, dx$ (b) $\int Sin^m x \ Cos \, nx \, dx$

4. Find the perimeter of the loop of the curve
 $9ay^2 = (x-2a)(x-5a)^2$.

5. (a) Evaluate $\lim_{n\to\infty} \left[\frac{1^2}{1^3+n^3} + \frac{2^2}{2^3+n^3} + ... + \frac{n^2}{n^3+n^3} \right]$.

 (b) Evaluate $\lim_{n\to\infty} \frac{[(n+1)(n+2)(n+3) ....(n+n)]}{n}$

6. Find the area between the curve $y^2(a+x) = (a-x)^2$ and its asymptote.

7. Find the volume of revolution of the loop of the curve $y^2(a+x) = x^2(a-x)$ about the x-axis.

8. Solve :– (a) $y = 2px + p^2$ (b) $y = px - x^4p^2$.

9. Solve the following differential equations :–

 (a) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4y = x^2$. (b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{2x}$

GROUP-B

10. (a) Define a convex set and a hyper plane and prove that a hyper plane is a convex set.

 (b) Prove that the intersection of a finite number of convex sets is a convex set.

11. Find the volume of the Tetrahedron, the co-ordinates of whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4).

12. (a) Find the equation of the right circular cylinder whose axis is given by $\frac{x}{1} = \frac{y}{0} = \frac{z}{2}$ and radius $\sqrt{7}$.

 (b) Find the equation of the sphere which passes through the point (•, •, •) and the circle $x^2 + y^2 + z^2 = a^2, z = 0$.

GROUP-C

13. What do you mean by Simple Harmonic Motion, derive an expression for time period.

14. State and prove principle of virtual work.

15. (a) State and establish the principle of energy.

 (b) Analyze the motion of a body under inverse square law.

16. Deduce the general conditions for equilibrium of a system of co-planar forces.

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Nalanda Open University
Annual Examination - 2020
B.Sc. Mathematics (Honours), Part-III
Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. Prove that (R^n, d) is complete where d on R^n is defined as $d(x, y) = \left[\sum_{i=1}^{n} \left|x_i - y_i\right|^2\right]^{\frac{1}{2}}$.

2. (a) Let (X, d) be a metric space and $A \subseteq X$ then show that A is closed if and only if $A \subseteq \overline{A}$.

   (b) If M and N are two subsets of a metric space (X, d) then show that $\overline{MUN} = \overline{M} \cup \overline{N}$.

3. (a) In a metric space (x,d) prove that the union of an arbitrary collection of open sets is open.

   (b) In a metric space (x,d) prove that any finite intersection of open sets in X is open.

4. (a) State and prove Minkowsky's in equality.

   (b) State and prove Cauchy Schwartz inequality.

5. (a) Define the convergence of a sequare (x_n) in a metric space (x,d) an prove that limit of sequence in (x,d) if it exists is unique.

   (b) Define a Cauchy sequence in a metric space (x,d) and prove that every convergent sequences in (x,d) is a Causchy sequence in (x,d).

Group 'B'

6. (a) Let (X,T_1) and (Y, T_2) be two Topological spaces then a function $f: X \to Y$ is $T_1 \to T_2$ continuous if and only if for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.

   (b) Let (X, T_1) and (Y, T_2) be two Topological spaces then a mapping $f: X \to Y$ is open if and only if $f(A^{\circ}) \subseteq [f(A)]^{\circ}$ for every subset A of X.

7. Let (X, T) be a Topological space and A and B are any two subsets of X and $\overline{A}$ denotes the closure of A then prove that :

   (a) $\overline{\phi} = \phi$ (b) $A \subseteq \overline{A}$ (c) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$

   (d) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (e) $(\overline{A \cap B}) \subseteq \overline{A} \cap \overline{B}$ (f) $\overline{\overline{A}} = A$

Group 'C'

8. (a) Prove that every bounded monotonic function $f:[a,b] \to R$ is R-integrable on [a,b].

   (b) If a function f is continuous on [a,b] then prove that it is integrable on [a,b].

9. (a) if f and g are bounded and R-integrable on [a,b] then prove that f+g is also bounded and R-integrable on [a,b] and $\int_a^b \{f(x) + g(x)\}dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.

   (b) if f and g are two bounded and R-integrable functions in [a,b] then prove that fg is bounded and R-integrable in [a,b].

Group 'D'

10. (a) Find the radius of convergence of the series $\sum \frac{n^n x^n}{n}$.

    (b) Prove that the series $\sum\left(\frac{\cos n\theta}{n^2}\right)$ is convergent for all real values of θ.

11. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n \log n (\log\log n)^p}$.

NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-III
PAPER–VI (Honours)
Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Define an automorphism of a group G. Let $x \in G$, then prove that the function f defined by $f(g) = x^{-1}gx$ for $g \in G$ is an automorphism of G.
   (b) If G is a group, then for every element $g \in G$, prove that $C_o(g)$ is a Subgroup of G.
2. Prove that the set of all polynomials in $Z[x]$ with constant term O is prime ideal in $Z[x]$.
3. Show that the union of two ideals is again an ideal.
4. Define the principal ideal ring and show that the ring of integers is a principal ideal ring.
5. Define a ring homomorphism. If $f : R \to R'$ be a homomorphism of a ring R onto a ring R' then show that f is a homomorphism iff Kernel of $f = \{0\}$.
6. Show that any ring can be embedded in a ring with unity.

GROUP 'B'

7. (a) Prove that $2^{No} = c$, where symbols have their usual meaning.
   (b) For cardinal numbers α, β, γ prove that
   (i) $\alpha^{\beta} \cdot \alpha^{\gamma} = \alpha^{\beta + \gamma}$ (ii) $(\alpha \cdot \beta)^{\gamma} = \alpha^{\gamma} \beta^{\gamma}$ (iii) $(\alpha^{\beta})^{\gamma} = \alpha^{\beta\gamma}$
8. (a) Introduce the concept of order types and construct the product of two order types.
   (b) If X is any non-empty set then show that card $(P(x))$ is 2 where $P(x)$ is the power set of X.
9. State and prove Cantor's Theorem.

GROUP 'C'

10. State and prove Cauchy integral formula.
11. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann differential equations are satisfied.
    (b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^2} dz$. Where C is the circle $|z| = 3$.
12. Obtain the necessary and sufficient condition for differentiability of a complex valued function.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–VII (Honours)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Prove that the set of all feasible solutions of a linear programming problem constitutes a convex set.

   (b) Define convex combination of vectors in R^n. Prove that the set of convex combinations of a finite number of linearly independent vectors $v_1, v_2, v_3, \ldots\ldots\ldots, v_n$ is a convex set.

2. (a) Define a convex set, the subset of R^n and show that the finite intersection of convex sets is a convex set.

   (b) Prove that every hyperplane is convex.

3. Use simplex method to solve :
   Maximize : $z = 3x_1 + 9x_2$
   Subject to $x_1 + 4x_2 \le 8, \quad x_1 + 2x_2 \le 4$ and $x_1 \ge 0, x_2 \ge 0$.

GROUP 'B'

4. Solve :

   (a) $pz - qz = z^2 (x + y)^2$

   (b) $(y + z) p + (z + x)q = x + y$

5. Solve by using Charpit's method $(p^2 + q^2)x = pz$.

6. Test for integrability and hence solve the equation
   $(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$

7. Use Monge's method to find the complete solution of the equation
   $2x^2r - 5xys + 2y^2t + 2(px + qy) = 0$

8. (a) Solve $\dfrac{dx}{x^2 - y^2 - z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$.

   (b) Solve $\dfrac{dx}{dt} + 4x + 3y = t$ and $\dfrac{dy}{dt} + 2x + 5y = e^t$.

GROUP 'C'

9. Find the attraction of a uniform sphere at an external point of it.

10. State and prove Laplace theorem in cartesian form.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–VIII (Honours)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Describe Newton-Gregory formula for backward interpolation.

2. (a) Discuss Newton-Raphson's method to obtain approximate value of root of $f(x) = 0$.
   (b) By using synthetic division solve
   $f(x) = x^3 - x^2 - (1.001)x + 0.9999 = 0$ in the neighbourhood of $x = 1$.

3. (a) Derive Simpson's $\frac{3}{8}$th rule for numerical integration.
   (b) Use Weddle's rule to evaluate $\int_0^{10} \frac{1}{x+1} dx$.

4. Applying analytical method for finding roots of an equation based on Rolle's theorem and demonstrate on $3x - \sqrt{1 + \sin x} = 0$.

5. Use Gauss-Jordan method to solve the system of equations
   $x_1 + 2x_2 + x_3 = 8,\ 2x_1 + 3x_2 + 4x_3 = 20$ and $4x_1 + 3x_2 + 3x_3 = 16$
   taking initial condition $x_1 = 0,\ x_2 = 0,\ x_3 = 0$.

6. (a) Describe Picard's method of successive approximation.
   (b) Apply Runge-Kutta mehtod for the solution of first order differential equation.

7. (a) Explain the meaning of the operators E and Δ. Show that E and Δ are commutative with respect to variables.
   (b) Evaluate $\Delta^3 (1 - x)(1 - 2x)(1 - 3x)$ and $\Delta^n \left(e^{ax + b}\right)$ where a and b are constants.

8. (a) State and prove Adam's predictor formula.
   (b) Describe Milne corrector formula.

9. (a) Explain Gauss's method of elimination for the solution of a system of m equations in m variables.
   (b) Solve the following system of equations

   $$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = 1$$

   $$\frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 = 0$$

   $$\frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 = 0$$

10. (a) Derive Trapezoidal and Simpson's one third rule to numerical integration.
    (b) Solve difference equation $\bigcup_{x+1} = 2^x \cup_x$.

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