NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–I
(Advanced Abstract Algebra)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove Kronecker's theorem.
2. What do you mean by extension of a field. Establish the transitivity property of finite extension of a field.
3. State and prove fundamental theorem of Galois theory.
4. State and prove Jordan-Holder theorem on any group.
5. Define Homomorphism and Kernel of homomorphism from a module M into a module N. If f is a module homomorphism then f is an isomorphism if and only if $K(f) = 0$. Prove this.
6. (a) Define algebraic and simple extension of a field and give an example of each one.
   (b) If a and b are algebraic over a field F then prove that $a + b$, ab, ab^{-1} $(b \neq 0)$ are also algebraic over F.
7. (a) Prove that in every principal ideal domain, each pair of elements has a greatest common divisor.
   (b) Prove that the range of homomorphism of a module is a sub-module of the module.
8. (a) Define a subnormal series of a group. Hence or otherwise form a subnormal series of the additive group of integers.
   (b) Construct all the composition series of Z_{60}.
9. (a) Show that a module M is the direct sum of two modules M_1 and M_2 if and only if (i) $M_1 + M_2$ and (ii) $M_1 \cap M_2 = \{0\}$ are sub modules.
   (b) Define a sub-module of a module M. Show that arbitrary intersection of sub-modules of a module M is a sub-module of M.
10. (a) Prove that if $K = \phi(\sqrt{2})$ where ϕ is the field of all rational numbers then ϕ is the fixed field under the group of automorphism of K.
    (b) Find the Galois group of the equation $x^3 - 2 = 0$ over the field Q of rational numbers.

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EXAMINATION PROGRAMME-2021
M.Sc. Mathematics, Part-I

Date	Papers	Time	Examination Centre
20.05.2022	Paper–I	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
24.05.2022	Paper–II	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
26.05.2022	Paper–III	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
28.05.2022	Paper–IV	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
30.05.2022	Paper–V	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
01.06.2022	Paper–VI	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
03.06.2022	Paper–VII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
06.06.2022	Paper–VIII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–II

(Real Analysis)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove Bolzano-Weierstrass theorem and give a suitable example of it.
   (b) Deduce Bolzano-Weierstrass theorem from Heine-Borel theorem.

2. (a) State and prove a necessary and sufficient condition for a function f to be R-integrable over [a, b].
   (b) If $f, g \in R(\alpha)$ on [a, b] then prove that $f+g \in R(\alpha)$ and $\int_a^b (f+g)\,d\alpha = \int_a^b f d\alpha + \int_a^b g d\alpha$.

3. (a) If $f \in R(\alpha)$ and α is monotonically increasing on [a, b], then show that $|f| \in R(\alpha)$ on *[a, b]* and $\left|\int_a^b f\,d\alpha\right| \le \int_a^b |f|\,d\alpha$.
   (b) If $f \in R(\alpha)$ on [a, b] and $\int_a^b f\,d\alpha = 0$ for every f which is monotonic on [a, b] then prove that α must be constant on [a, b].

4. (a) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{\lfloor n}{n^n} x^n$.
   (b) State and prove Abel's theorem.

5. If $f : R^2 \to R$ be defined by $f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}$, $(x, y) \ne (0, 0)$ and $f(0, 0) = 0$ then show that $D_{1,2}\, f(0, 0) \ne D_{2,1}\, f(0, 0)$.

6. Prove that a necessary and sufficient condition for a function f on [a, b] to be of bounded variation is that it can be written as the difference of two monotonically increasing functions on [a, b].

7. State and prove implicit function theorem.

8. State and prove inverse function theorem.

9. What do you mean by the Extreme values of a function in the case of a function of n-variables and find these values in the case of the function defined by $f(x, y, z) = 2xyz - 4zx - 2yz + x^2 + y^2 + z^2 - 2x - 4y + 4z$.

10. Find $\frac{\partial(y_1, y_2, y_3, ---, y_n)}{\partial(x_1, x_2, x_3, ---, x_n)}$ where

$y_1 = x_1(1 - x_2)$
$y_2 = x_1 x_2 (1 - x_3)$
$y_3 = x_1 x_2 x_3 (1 - x_4)$

$y_{n-1} = x_1 x_2 x_3 --- x_{n-1}(1 - x_n)$
$y_n = x_1 x_2 x_3 --- x_n$

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–III

(Measure Theory)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that the measure of a Denumerable set is Zero.
   (b) If A, B are L-measurable subsets of R^k then prove that $A \cup B$, $A \cap B$ are also L-measurable subsets of R^k.

2. (a) If (S_r) is a Sequence of L-measurable subsets of R^k then show that $\bigcup_{r=1}^{\infty} S_r$ is also L-measurable.
   (b) If (A_n) is sequence of L-measurable subsets of R^k such that $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots\ldots\ldots \supseteq A_n \supseteq A_{n+1} \ldots\ldots\ldots$ and $A = \bigcap_{n=1}^{\infty} A_n$ and $m(A_1) < \infty$ then show that A is L-measurable and $m(A) = \underset{n \to \infty}{lt}\ m(A_n)$.

3. (a) Show that the class of all measurable functions is closed with respect to all algebraic operations.
   (b) If f is a measurable function then show that $|f|$ is also a measurable function.

4. If (f_n) is a sequence of measurable functions then show that the class of all measurable functions is closed with all analytic operations.

5. Give the analytic description of Cantor's Ternary set and show that it is an uncountable set of measure Zero.

6. State and prove Fatou's Lemma.

7. Define the Lebesque integral of function in details. If f and g are L-integrable then show that $\int (f+g)\,d\mu = \int f\,d\mu + \int g\,d\mu$.

8. State and prove dominated convergence theorem.

9. State and prove Lebesque monotone convergence theorem.

10. (a) Verify bounded convergence theorem for $f_n(x) = \dfrac{nx}{1+n^2x^2}$ $(0 \le x \le 1)$, $n = 1, 2, 3, --$.
    (b) Examine the L-integrability of $f(x) = \left(x^2 \sin\dfrac{1}{x^2}\right)$ over [0, 1].

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–IV

(Topology)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Define hereditary and topological properties and show that the property of a T_1-space is both hereditary and topological.
   (b) Prove that a topological space (X, T) is T_0 -Space if and only if $x, y \in X$ and $x \neq y \Rightarrow \overline{\{x\}} \neq \overline{\{y\}}$.

2. (a) Prove that a topological space (X, T) is normal space if and only if each neighbourhood of a closed set F contains the closure of some neighbourhoods of F.
   (b) Show that every metric space is a normal space.

3. (a) Prove that every compact subspace of the real line is closed and bounded.
   (b) What do you mean by a regular space. Prove that a compact Hausdorff space is regular.

4. (a) Prove that a finite sub-set of T_1-space has no cluster point.
   (b) Give an example of topological space which is a T_1-space but not a T_2-space.

5. Prove that an arbitrary intersection of topological spaces is a topological space.

6. If X and Y are topological spaces, then prove that X × Y is connected *iff* X and Y are connected.

7. (a) Prove that every compact subspace of a Hausdorff space is closed.
   (b) Define T_3-space and T_4-space and prove that every T_4-space is a T_3-space.

8. (a) Show that connectedness is not hereditary property.
   (b) Introduce the concept of connected and disconnected spaces and show that a topological space X is connected *iff* ϕ and X are its only subsets which are both open and closed.

9. (a) Show that the open interval (0, 1) on the real line R is not compact.
   (b) If (X, T) be a topological space and $A \subseteq X$, $B \subseteq X$ then show that $Int(A \cap B) = Int(A) \cap Int(B)$

10. (a) Prove that in a Hausdorff space every convergent sequence has a unique limit.
    (b) Let (X, T) be a topological space and $A \subseteq X$. Then show that
    (i) $(Int\ A)' = \bar{A}'$, (ii) $(\bar{A})' = Int(A')$

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–V
(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Prove that a linear operator E is a projection on some subspace *iff* it is an idempotent.

2. Let $V(F)$ be a finite dimensional vector space and W is a subspace of V, then show that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

3. (a) If f is a linear functional on a vector space $V(K)$ then show that (i) $f(0) = 0$ and $f(-x) = -f(x)$.
   (b) Prove that two real quadratic forms are equivalent iff they have the same rank and index.

4. (a) If R is a ring and L is a lattice of all ideals of R, then prove that L is a modular.
   (b) Prove that a partially ordered set $(P(X), \subseteq)$ is a lattice.

5. (a) If B is a Boolean algebra then prove that for $\forall x, y \in B$ the following are equivalent.
   (i) $x \wedge y' = 0$ (ii) $x \vee y = y$ (iii) $x' \vee y = 1$ (iv) $x \wedge y = x$
   (b) Show that the relation precedes $(x \leq y)$ in a Boolean algebra B is a partial order relation.

6. (a) Define isomorphism between two lattices. Give one example.
   (b) Prove that a necessary and sufficient condition for a one to one and onto mapping f between two lattices to be isomorphism is that f and f^{-1} are both order preserving.

7. (a) Prove that in Boolean Algebra, the complement of an element is unique.
   (b) Prove that a Boolean Algebra B is a complemented distributive lattice.

8. Convert $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$ to Jordan canonical form.

9. (a) Let {(1, 1, 1), (0, 1, 1), (0, 0, 1)} be a basis of Euclidean space R^3, then find its orthonormal basis.
   (b) Show that the matrix, $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent of index 3.

10. Define a linear transformation and its null space. If $U(f)$ and $V(f)$ are two vector spaces and T is a linear transformation from U into V, then show that the kernel T or null space of T is a subspace of U.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VI
(Complex Analysis)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that $u = \frac{1}{2}\log\left(x^2 + y^2\right)$ is a harmonic function. Also find the analytic function $f(z)$ whose real part is *u*.

   (b) Find the necessary and sufficient condition for analyticity of the function $f(z)$.

2. Find Taylor's expansion of the function $f(z) = \dfrac{z}{z^2 + 9}$ around $z = 0$.

3. (a) What is the pole of a function ? Also introduce the residue at simple pole and pole of order *m*.

   (b) Describe different kinds of singularities.

4. (a) Describe the radius of convergence of the power series $\sum\limits_{n=0}^{\infty} \dfrac{z^n}{2^n + 1}$.

   (b) State and prove Cauchy-Hadmard theorem for power series.

5. By introducing Bilinear transformation, derive the existence of fixed points of a Bilinear transformation.

6. (a) State and prove the necessary and sufficient condition for the transformation $w = f(z)$ to be conformal.

   (b) Show that the transformation $w = \dfrac{5 - 4z}{4z - 2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the w–plane and hence find its centre.

7. State and prove Cauchy's theorem.

8. Using Cauchy's integral formula evaluate $\int\limits_C \dfrac{z\,dz}{(9 - z^2)(z + 1)}$, where *C* is the circle described anticlockwise and having equation $|z| = 2$.

9. State and prove Poisson's integral formula.

10. Evaluate the following integrals :—

    (a) $\displaystyle\int_0^{\infty} \frac{dx}{(1 + x^2)^2}$

    (b) $\displaystyle\int_0^{2\pi} \frac{d\theta}{1 + a\,Cos\theta}$ *where* $a^2 < 1$

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VII
(Theory of Differential Equations)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Define Lipschitz condition in a region. Show that the following function do not satisfy the Lipschitz condition in the region indicated $f(x, y) = \frac{\sin y}{x}$, $f(0, y) = 0$, $|x| \le 1$, $|y| < \infty$.

2. State and prove Picard's-Lindelof theorem.

3. (a) Find e^A if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.

   (b) Determine the constants M *and* C and x for the initial value problem $y' = y$, $y(0) = 1$, $R = \{(x, y) : |x| \le 1 \text{ and } |y - 1| \le 1\}$.

4. (a) Compute the first three successive approximations for the solution of the equation $y' = y^2$; $y(0) = 1$.

   (b) Find an interval I containing Γ and a solution g of $y' = \frac{dy}{dx} = f(x, y)$ on I satisfying $g(\Gamma) = s$.

5. Solve by matrix method the system of equations $\frac{dx_1}{dt} = 9x_1 - 8x_2$; $\frac{dx_2}{dt} = 24x_1 - 8x_2$, where $x_1(0) = 1$ and $x_2(0) = 0$.

6. Prove that a necessary and sufficient condition that a solution matrix G be a fundamental matrix is that $G(x) \ne 0$ for $x \in I$.

7. (a) Determine the type and stability of the critical point (0, 0) of the non-linear system $\frac{dx}{dt} = Sinx - 4y$; $\frac{dy}{dt} = Sin2x - 5y$.

   (b) Explain the nature of critical point of a non-linear system $\frac{dx}{dt} = ax + by + \phi(x, y)$ and $\frac{dy}{dt} = cx + dy + \psi(x, y)$.

8. (a) Explain different type of critical points for a system and give the geometrical meaning of each critical point.

   (b) Find the nature of the critical point (0, 0) of the system $\frac{dx}{dt} = x + 5y$, $\frac{dy}{dt} = 3x + y$ and discuss their stability.

9. (a) Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a +ve integer.

   (b) Derive an expression for the generating function for Bessel's function.

10. Find the Rodrigue's formula for Legendre polynomial.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VIII
(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Define a countable set. Prove that [0, 1] is uncountable.
   (b) If A and B are two countable sets then show that A × B is also countable.
2. (a) Prove that $2^{N_0} = C$, where N_0 is the cardinal number of the set N and C is the cardinal number of [0, 1].
   (b) State and prove Schroder-Bernstein theorem.
3. (a) For any three cardinal number α, β, γ; show that (i) $\alpha^{\beta}\alpha^{\gamma} = \alpha^{\beta+\gamma}$, (ii) $(\alpha\beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$.
   (b) State Axiom of choice and Zermelo's postulates. Show that Axiom of choice is equivalent to Zermelo's postulates.
4. (a) Prove that a pseudograph is Eulerian iff it is connected and every vertex is even.
   (b) What do you mean by a complete graph. Show that a complete graph of n vertices is a planner if $n \le 4$.
5. (a) Determine the difference between a circuit and Eulerian circuit.
   (b) Define isomorphism between two graphs and give two examples of isomorphic graphs.
6. (a) Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph.
   (b) If g is a connected graph with e-edges and v-vertices, then prove that $e \le 3v - 6$.
7. (a) State and prove the division algorithm of integers.
   (b) State and prove Chinese remainder theorem.
8. (a) Define congruency between two integers under a positive integer m. Prove that the relation $a \equiv b \pmod{m}$ defines an equivalence relation on the set of integers.
   (b) Show that $(a, m_1) = 1, (a, m_2) = 1 \Leftrightarrow (a, m_1 m_2) = 1$.
9. (a) Prove that $\left[\vec{r}^{\,I}, \vec{r}^{\,II}, \vec{r}^{\,III}\right] = \dfrac{T}{\rho^2}$. Where $\vec{r}$ is the current point, T is torsion and ρ is the radius of curvature.
   (b) What is a circular helix ? Find the osculating plane at the point $P(\theta)$ on the helix $x = a\, Cos\theta,\ y = a\, Sin\,\theta,\ z = c\theta$.
10. (a) If $x \equiv a \pmod{7} \equiv b \pmod{11} \equiv c \pmod{13}$, then prove or disprove that $x \equiv -286a + 364b - 77c \pmod{1001}$.
    (b) State and prove Fermat's theorem.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
Calculator is Allowed.

1. (a) Compare Newton's method with Regula-Falsi method. Apply Newton's Raphson method to find square root of 12 to five places of decimals.
   (b) Prove that $(1+\Delta)(1-\nabla)=1$.
2. Define factorial notation and prove that $(x)^{(-n)}=\dfrac{1}{(x+hn)^{(n)}}$. Where h is the interval of differencing.
3. Determine the value of the integral $\int_4^{5.2}\log x\,dx$ by Trapezoidal method.
4. Solve the equation,
   $y_{x+3}-y_{x+2}-y_{x+1}-y_x=0$, where $y_0=2,\ y_1=-1,\ y_2=3$.
5. Form the difference equation corresponding to the family of curves $y_x=ax^2+bx-3$.
6. Find the formula for Quadrature for equally spaced arguments and hence derive Simpson's three-eighth rule.
7. (a) Find f(6) when f(0) = 3, f(1) = 6, f(2) = 8, f(3) = 12 and the third difference being constant.
   (b) Find the positive root of $xe^x-1=0$ lying between 0 and 1 using iteration method.
8. (a) Find all the real roots of the equation $x^2+4\,Sin\,x=0$ correct to four places of decimals.
   (b) Obtain the missing term in the following table :—

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
f(x)	0.135	?	0.111	0.100	?	0.082	0.024

9. Compute $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at $x=2.03$ by Newton's Backward difference formula using the following table :—

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

10. Form Gauss's central difference table and apply it to determine e^{117} from the table :—

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

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EXAMINATION PROGRAMME-2021
M.Sc. Mathematics, Part-II

Date	Papers	Time	Examination Centre
26.07.2022	Paper–IX	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
28.07.2022	Paper–X	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
30.07.2022	Paper–XI	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
01.08.2022	Paper–XII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
03.08.2022	Paper–XIII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
05.08.2022	Paper–XIV	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
06.08.2022	Paper–XV	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
08.08.2022	Paper–XVI	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–X

(Functional Analysis)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
Calculator is Allowed.

1. State and prove Hahn-Banach theorem.
2. Sate and prove F. Riestz's theorem.
3. Define a normed linear space and a Banach space. In a normed linear space prove that $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.
4. (a) Let X and Y be two normed linear spaces where X is finite dimensional. Then show that every linear map from X to Y is continuous.
   (b) Prove that $x_n \to x$ *w.r.t.* $\| \ \|$ if and only if $x_n \to x'$ *w.r.t.* $\|x\|'$.
5. (a) Let L be a linear space over F, then show that the sum of two inner products on L is also an inner product on L.
   (b) If M and N are closed linear sub spaces of a Hilbert space H such that $M \perp N$ then prove that the linear sub space M + N is also closed.
6. State polarization identity and explain about it in an inner product space.
7. Give an example of a Banach space which is not a Hilbert space.
8. If $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, then prove that dual of l_p is l_q.
9. (a) If H is a Hilbert space, then show that the conjugate space H* is also a Hilbert space.
   (b) If a Hilbert space H is separable, then show that every orthonormal set of H is countable.
10. (a) If the mapping $T \to T'$ is norm preserving mapping of $\beta(N)$ to $\beta(N')$ then prove that,
    (i) $(\alpha T_1 + \beta T_2)' = \alpha T_1' + \beta T_2'$, and
    (ii) $(T_1 T_2)' = T_2' T_1'$.
    (b) If T is a continuous linear transformation of a Banach space X into Banach space Y, then show that T is an open mapping.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XI
(Partial Differential Equations)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. By using the method of separation of variables solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

2. Find the general solution of the partial differential equation $px(x+y) - qy(x+y) = (x-y)(2x+2y+z)$.

3. Explain Charpit's method for the solution of non-linear partial differential equation of the first order.

4. Solve,
   (a) $(D - D'^2)z = Cos\,(x - 3y)$
   (b) $(D^2 - DD' + D' - 1)z = Cos\,(x + 2y) + e^y$.

5. Using Charpit's method solve the following partial differential equations :—
   (a) $2zx - px^2 - 2qxy + pq = 0$
   (b) $(p^2 + q^2)y = qz$

6. (a) Solve the partial differential equation $xyr + x^2 s - yp = x^3 e^y$.
   (b) Describe Jacobi's method to solve the partial differential equation $F(x,\ y,\ z,\ p,\ q) = 0$.

7. Reduce $yr + (x+y)s + xt = 0$ to canonical form and hence solve it.

8. (a) Derive the Fourier equation of heat conduction.
   (b) A rod of length ℓ with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to *0°c* and are kept at that temperature. Find the temperature *u (x, t).*

9. (a) Solve the boundary value problem $\frac{\partial u}{\partial x} = u\frac{\partial u}{\partial y}$, when $u(0,\ y) = 8e^{-3y}$.
   (b) Show that the family of surfaces defined by $x^2 + y^2 =$ constant, is a family of equi-potential surfaces in free space and hence find the law of potential.

10. Solve the boundary value problem $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le t,\ t > 0$ subject to the boundary conditions $\left.\begin{array}{l} u(0,t) = 0,\ t > 0 \\ \frac{\partial u}{\partial x}(\ell, t) = 0,\ t > 0 \end{array}\right\}$ and the initial conditions $u(x,0) = \begin{cases} x, & 0 \le x < \frac{\ell}{4} \\ \frac{\ell}{2} - x; & \frac{\ell}{4} \le x < \frac{\ell}{2} \\ 0; & \frac{\ell}{2} \le x < \ell \end{cases}$ and

$\frac{\partial u}{\partial t}(x, 0) = 0,\ 0 \le x < \ell$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XII
(Analytical Dynamics)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Give the physical significance of Hamilton characteristic function.
   (b) Derive Hamilton-Jacobi equation and then find Hamilton's characteristic function.
2. (a) Prove that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical. Find the generating function $F(q, Q)$.
   (b) Prove that Lagrange's Bracket does not obey the commutative law of algebra.
3. Find the equation of motion of simple pendulum applying Lagrange's equation of motion.
4. Discuss the motion of a sphere when the small sphere rolls without slipping on the rough interior of a fixed vertical cylinder of greater radius.
5. (a) Using invariance of Bilinear form show that the transformation $Q = \frac{1}{p}$ and $P = p^2 q$ is canonical.
   (b) Describe the motion of particle about revolving axes.
6. State and prove Jaicobi-Poisson theorem.
7. Derive Lagrange's equation of motion from Hamilton's canonical form of equations.
8. Derive the formula for kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of kinetic energy.
9. What do you mean by Hamilton's function ? Find the differential equations for Hamilton's function.
10. Discuss the motion of spherical pendulum deducing from Hamilton's canonical equations of motion.

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M.Sc. Mathematics, Part-II

PAPER–XIII

(Fluid Mechanics)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Derive the equation of continuity in Cartesian form.

2. Derive Euler's equation of motion in cylindrical polar co-ordinates.

3. Derive the equation of motion under impulsive force.

4. Describe the motion of a fluid between rotating co-axial circular cylinders.

5. A velocity field is given by $\vec{q} = \dfrac{x\,\vec{j} - y\,\vec{i}}{x^2 + y^2}$; calculate the circulation round the square having corners at (1, 0), (2, 0), (2, 1) and (1, 1). Also test for the flow of rotation.

6. Prove that the fluid motion is possible when velocity at (x, y, z) is given by $u = \dfrac{3x^2 - r^2}{r^5}$, $v = \dfrac{3xy}{r^5}$, $w = \dfrac{3xz}{r^5}$.

7. (a) What do you mean by Source, Sink and Doublet. Describe them with suitable examples of each.

   (b) A velocity field is given by $\vec{q} = -x\,\hat{i} + (y + t)\,\hat{j}$. Find the stream function and the stream lines for field at $t = 2$.

8. (a) Derive the rate of strain tensor of fluid in motion.

   (b) Show that the velocity field defined at a point P by $(1+2y-3z,\ 4-2x+5z,\ 6+3x-5y)$ represents a rigid body rotation.

9. Derive Navier-Stokes equation of motion of viscous fluid.

10. (a) Write notes on the following :–

    (i) Velocity Potential

    (ii) Velocity Vector

    (iii) Boundary Surface

    (b) The velocity $\vec{q}$ in a three dimensional flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{i} - y\,\hat{j} - z\,\hat{k}$. Determine the equations of streams passing through the point (1, 1, 1).

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M.Sc. Mathematics, Part-II

PAPER–XIV

(Operation Research)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Find the dual of the following L.P.P.
   Min $z = x_1 + x_2 + x_3$
   Such that $x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \le 3$, $2x_2 - x_3 \ge 4$; $x_1, x_2 \ge 0$ *and* x_3 is unrestricted in sign.
2. (a) Show that every extreme point of the convex set of feasible solution is a B.F.S. (Basic Feasible Solution).
   (b) Define a convex set in R^n. Let S and T be two convex sets in R^n then show that for any scalars K_1 and K_2, $K_1S + K_2T$ is also a convex set in R^n.
3. (a) Solve the following L.P.P. by any method of your choice (except graphically)
   Max $z = 5x_1 + 7x_2$ s.t.
   $x_1 + x_2 \le 4$, $3x_1 + 8x_2 \le 24$, $10x_1 + 7x_2 \le 35$ *and* $x_1, x_2 \ge 0$.
   (b) If (1, 2, 3) is a feasible solution of the set of equations $4x_1 + 2x_2 - 3x_3 = 1$; $6x_1 + 4x_2 - 5x_3 = 1$ then reduce the F.S. to B.F.S. of the set.
4. Solve the following L.P.P. by using two phase simplex method
   Min $z = x_1 + x_2$
   Subject to $2x_1 + x_2 \ge 4$, $x_1 + 7x_2 \ge 7$; $x_1, x_2 \ge 0$.
5. Solve the following L.P.P. problem by simplex method.
   Minimize $z = x_1 - 3x_2 + 2x_3$
   Subject to $3x_1 - x_2 + 2x_3 \le 7$, $-2x_1 + 4x_2 \le 12$, $-4x_1 + 3x_2 + 8x_3 \le 10$; $x_1, x_2, x_3 \ge 0$.
6. (a) Prove that dual of the dual of a given primal is the primal itself.
   (b) If X_0 and W_0 are feasible solutions to the primal and dual respectively then prove that $c\,X_0 \le W_0 b$.
7. Solve the following NLPP using the method of Lagrangian multipliers
   Min $z = x_1^2 + x_2^2 + x_3^2$
   Subject to constraints $4x_1 + x_2^2 + 2x_3 = 14$; $x_1, x_2, x_3 \ge 0$.
8. Solve the following assignment problem represented by the following matrix.

	I	II	III	IV	V	VI
A	9	22	58	11	19	97
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	40	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

9. Solve the following L.P.P.
   Max $z = 10x_1 + 3x_2 + 6x_3 + 5x_4$
   S.t. $x_1 + 2x_2 + x_4 \le 6$, $3x_1 + 2x_3 \le 5$, $x_2 + 4x_3 + 5x_4 \le 3$ *and* $x_1, x_2, x_3, x_4 \ge 0$
   Also, compute the limits for a_{11} and a_{23} so that the new solution remains optimal feasible solution.
10. The pay-off matrix of a game is given below. Find the solution of the game for A and B.

		B				
		I	II	III	IV	V
	I	-2	0	0	5	3
A	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XV

(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that the co-variant derivative of a co-variant vector is a mixed tensor of rank two.
   (b) Prove that the outer product of two tensors (r, s) and (p, q) types is a tensor of $(r + s)(p + q)$ type.

2. (a) State and prove quotient theorem of tensors; give an example.
   (b) What do you mean by symmetric and skew symmetric tensors. Prove that a symmetric tensor of rank two has at most $\frac{1}{2} N(N+1)$ different components in V_N. Where as a skew symmetric tensor of rank two has $\frac{1}{2} N(N-1)$ independent components in V_N.

3. (a) In the matrix notation express the following transformation equations for (i) a covariant vector, (ii) a contravariant vector, (iii) a contravariant tensor of rank two assuming N = 3.
   (b) If a covariant tensor has components xy, $2y - z^2$, zx in rectangular co-ordinates then determine its covariant components in spherical co-ordinates.

4. (a) If $L\{F(t)\} = f(s)$ then prove that

   $$L\{F^n(t)\} = s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) \ldots\ldots\ldots\ldots - s F^{n-2}(0) - F^{n-1}(0).$$

   (b) Prove that the Laplace transform of $\frac{Sin\, at}{t}$ is $Cot^{-1}\left(\frac{s}{a}\right)$.

5. (a) State and prove Convolution theorem for inverse Laplace transform.
   (b) Find the inverse Laplace transform of the following

   (i) $\frac{5s + 3}{(s - 1)(s^2 + 2s + 5)}$ (ii) $\frac{1}{s^2(s^2 + a^2)}$

6. Solve $(D^3 - D^2 + 4D - 4)x = 68 e^t\, Sin\, 2t$. Using Laplace tranform.

7. Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \ge a \end{cases}$ and hence evaluate

   (i) $\int_{-\infty}^{\infty} \frac{Sin\, sa\ Cos\, sx}{s} ds$ (ii) $\int_{0}^{\infty} \frac{Sin\, s}{s} ds$

8. (a) Define Fredholm integral and Voltera integral equations.
   (b) Prove that the function $u(t) + (1 + x^2)^{-1/2}$ is a solution of the voltera integral equation.

   $$u(x) = \frac{1}{1 + x^2} - \int_0^x \frac{t}{1 + x^2} u(t) dt.$$

9. Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} + e^x y = x$ with the initial conditions $y(0) = 1$ and $y'(0) = -1$.

10. Determine the deterministic model with instantaneous production. Shortage allowed.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XVI
(Programming in 'C')
Annual Examination, 2021

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Explain different types of statements written in C with examples.
2. Describe different data types used in C programming with examples.
3. Define reserved words in C ? What is the difference between the expression "++a" and "a++" ?
4. What is recursion ? Write a program in C using recursion.
5. Explain some of the looping statements with examples.
6. What is an Array ? How does an Array differ from an ordinary variable ? Write a program in C using array.
7. What are different types of **if and else** statements used in C ? Explain each of them with help of examples.
8. What are structures ? When and why are they used in C ? Give an example to explain them.
9. What are logical errors and how does it differ from syntax errors ? Write a program in C to swap the value of two variables
10. Write short notes on any ***Two*** of the following :—
    (i) Constant and Variables in C
    (ii) Switch statement
    (iii) Operators
    (iv) Pointers in C.

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M.Sc. Mathematics, Part–II, Paper–XVI
Practical Counselling & Examination Programme, 2021

A. Counselling Class Programme - ONLINE

अनुक्रमांक	*तिथि*	*समय*	*स्थान*
All New & Old Students	11.08.2022 एवं 13.08.2022	12.00 Noon to 3.00 PM	**ONLINE** (Online Counselling Class, Microsoft Teams Platform पर कराया जायेगा, इसलिए वे Link से जुड़ने से पहले Play Store से Microsoft Teams Install कर लें, तदुपरान्त Name और Enrollment No. दर्ज कर, ससमय परामर्श कक्षा में शामिल होवें । Counselling Class का Link नालन्दा खुला विश्वविद्यालय के वेबसाईट www.nou.ac.in के Student Corner Section में दिया गया है ।)
All New & Old Students	16.08.2022	12.00 Noon to 3.00 PM	**OFFLINE** School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001

B. Practical Examination Programme - OFFLINE

अनुक्रमांक	*परीक्षा की तिथि*	*परीक्षा का समय*	*परीक्षा का स्थान*
All Old Batch Students & 190290001 to 190290105	17.08.2022	11.30 AM to 1.30 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
190290106 to 190290400	17.08.2022	2.30 PM to 4.30 PM	
190290401 to 190290720	18.08.2022	11.30 AM to 1.30 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
190290721 to 190291084	18.08.2022	2.30 PM to 4.30 PM	

इस कार्यक्रम में किसी भी परिस्थिति में परिवर्तन नहीं होगा ।