Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Honours), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) What do you mean by a partially ordered set. If X is any non-empty set then show that $(P(X), \subseteq)$ is a partially ordered set.
   (b) What do you mean by a Lattice and a complete Lattice, Give one example of each.
2. (a) What do you mean by a denumerable set. Prove that every infinite set has a denumerable subset.
   (b) If $f: X \to Y$ and $A \subseteq X$, $B \subseteq X$ then show that $f(A \cap B) \subseteq f(A) \cap f(B)$.
3. (a) Show that the set R of all real numbers is uncountable.
   (b) Show that a countable union of countable sets is countable.
4. (a) Define on equivalence relation on a non-empty set A and if R_1 and R_2 are any two equivalence relations on A then show that $R_1 \cap R_2$ is also an equivalence relation on A.
   (b) Define the Cartesian product of two non-empty sets A and B and prove that if A, B, C are any three non--empty sets then
   (i) $A \times (B - C) = A \times B - A \times C$ and
   (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Group - B

5. (a) Find the eigen values of the matrix. $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$.
   (b) Prove that $A(\text{adj } A) = (\text{adj } A)A = |A|$ where A is n-rowed square matrix.
6. Show that $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I = 0$. Hence or otherwise find the inverse of A.
7. Find the rank of the matrix $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$.
8. Solve by matrix method the following simultaneous equations.
   $x + y + z = 6, \quad 2x + y - 3z = -5, \quad 3x - 2y + z = 2$

Group - C

9. (a) Prove that the intersection of two subgroups of a group G is also a subgroup of that group.
   (b) State and prove Lagranges Theorem.
10. (a) Prove that any two left or right cosests of a subgroup of a group G are either disjoint or identical.
    (b) Prove that if for every element 'a' in a group G, $a^2 = e$ then G is an abelian group.
11. (a) State and prove Gregories series for expansion of $\tan^{-1}x$ in ascending powers of x.
    (b) Find the expansion of $\sin\theta$ is ascending powers of θ.
12. (a) Find the condition that the cubic $x^3 - px^2 + qx + r = 0$ should have its roots be in Harmonic progression.
    (b) The equation $3x^4 - 25x^3 + 50x^2 - 50x + 12 = 0$ has two roots whose product is $2i$, find all the roots.

Nalanda Open University
Annual Examination - 2020
B.Sc. Mathematics (Honours), Part-I
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) State and prove Euler's theorem for Homogeneous function of two independent variables x and y of degree n.

   (b) If $v = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \sin 2v$.

2. Evaluate the following limits.

   (a) $\underset{x\to 0}{Lt}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$ (b) $\underset{x\to \frac{\pi}{2}}{Lt}(\sin x)^{\tan x}$.

3. (a) State and prove Maclaurin's theorem.

   (b) Prove that $\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \ldots\ldots\infty$

4. (a) State and prove Leibnitz's theorem.

   (b) If $y = \sin(m \sin^{-1} x)$ then prove that :

   $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$

5. (a) Find the pedal equation of the curve $r^n = a^n \sin(n\theta)$.

   (b) Show that the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

   touches the curve $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$.

Group - B

6. Evaluate any two of the following:

   (a) $\int(\sqrt{\tan x} + \sqrt{\cot x})dx$ (b) $\int\sqrt{\sec x + 1}\, dx$ (c) $\int\frac{d\theta}{(a^2\sin^2\theta + b^2\cos^2\theta)^2}$

7. Evaluate

   (a) $\int_0^{\pi} x\log(\sin x)dx$ (b) $\int_0^1 \frac{\log(1+x)}{(1+x^2)}$

8. (a) Evaluate $\underset{r\to\infty}{lt}\sum_{r=1}^{n}\frac{r^3}{r^4+n^4}$.

   (b) If $I_{m,n} = \int \overset{m}{\cos} x \overset{n}{\sin} x\, dx$ then show that $(m+n)\, I_{m,n} = \cos^{m-1} x \cdot \sin^{n+1} x + (m-1)I_{m-2,n}$

9. Find the volume formed by the revolution of the loop of the curve $y^2(a-x) = x^2(a+x)$.

10. Find the area between the curve $x(x^2+y^2) = a(x^2-y^2)$ and its asymptote. Also find the area of the loop.

Group – C

11. (a) Prove that the spheres

    $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ cut each other orthogonally.

    (b) Show that $3x^2+4y^2+5z^2-6yz-4zx-2xy=0$ represents a pair of planes.

12. (a) Find the equation of the plane cutting off intercepts a, b, c from the axes.

    (b) Find the angle between two lines whose direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) are given.

Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Subsidiary), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting atleast **One** question from each group.*
All questions carry equals marks.

Group - A

1. (a) Define the Cartesian product of two non-empty sets A and B. If A, B, C are three non-empty sets then prove that $(A - B) \times C = A \times C - B \times C$.
   (b) Define Reflexive, Symmetric and Transitive relations giving one example of each.
2. (a) What do you mean by an equivalence relation on a set A. If R_1 and R_2 are two equivalence relations on A then show that $R_1 \cap R_2$ is also an equivalence relation on A.
3. Prove that the set Pn of all permutations on n symbols is a finite non-abelian group of order n with respect to composition of mappings as the operation.
4. (a) If G is group then prove that $(ab)^{-1} = b^{-1}a^{-1} \ \forall\ a, b \in$ G.
   (b) If a and b are any two elements of a group G then prove that the equation $ax = b$ and $ya = b$ have unique solution in G.

Group - B

5. State and prove De-Moivre's theorem.
6. Reduce $(\alpha + i\beta)^{x + iy}$ in the form of $A + iB$.
7. Find $(1+i)^{\frac{1}{3}}$?
8. Show that the sequence (x_n) where
   $x_1 = 1, x_n = \sqrt{2 + x_{n-1}}$ is convergent and it converges to 2.
9. Prove that every convergent sequence is bounded.
10. Prove that a monotonic increasing sequence which is bounded above is convergent.
11. Prove that the sequence whose n^{th} term is $\left(\sqrt{n+1} - \sqrt{n}\right)$ is convergent.

Group - C

12. If $y = \tan^{-1}x$ then prove that:
    $(1 + x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.
13. Apply Maclaurin's series to expand $e^{x \sec x}$ as for as the term containing x^3.
14. Evaluate:
    (a) $\underset{x \to 0}{Lt} (\cot x)^{\left(\frac{1}{\log x}\right)}$ (b) $\underset{x \to 0}{Lt} \dfrac{x \cos x - \log(1+x)}{x^2}$.
15 (a) Prove that: $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$.
   (b) Give the geometrical meaning of scalar triple product of three vectors $\vec{a}, \vec{b}, \vec{c}$.
16. (a) If the point $(at_1^2, 2at_1)$ is one extremity of a focal chord of the parabola $y^2 = 4ax$ then find the co-ordinates of the other extremity and hence show that the length of the chord is $a\left(t_1 + \dfrac{1}{t_1}\right)^2$.
    (b) Prove that the two spheres:
    $S_1 \equiv x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and
    $S_2 \equiv x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$
    cut each other orthogonally if $2(u_1u_2 + v_1v_2 + w_1w_2) = d_1 + d_2$.

Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Honours), Part-II, Paper-III

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.
All questions carry equal marks.

Group-A

1. (a) Prove that a set E in R is compact if and only if E is closed and bounded.
   (b) Prove that Int. (A) is an open set.
2. (a) State and prove Heine-Borel theorem.
   (b) State and prove Bolzano Weierstrass theorem.
3. (a) Prove that every closed subset of a compact set in R is compact.
   (b) Prove that every compact subset of R is closed.

Group-B

4. (a) State and prove Logarithmic ratio test.
   (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n}$.
5. (a) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$.
   (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$.
6. (a) Prove that every monotonically decreasing sequence which is bounded tends to its greatest lower bound.
   (b) Prove that every bounded monotonically increasing sequence converges to its least upper bound.
7. (a) If (x_n) is a sequence where $x_n = \left(\sqrt{n+1} - \sqrt{n}\right)$ for all $n \in N$ then show that it is convergent and find its limit.
   (b) Prove that every Cauchy Sequence of real numbers is convergent.

Group-C

8. (a) Define the eigen values and eigen vectors of a square matrix and compute the eigen values and eigen vectors of the matrix A = $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
   (b) Prove that the set $(1, i, o), (2i, 1, 1), (1, 1 + i, 1 - i)$ is a basis for $V_3(C)$.
9. (a) Prove that $T : V_2(R) \to V_3(R)$ defined by $T(a, b) = ((a + b), (a - b), b)$ is a linear transformation.
10. If W_1 and W_2 are two subspaces of a finite dimensional vector space V over a field F then show that dim. $(W_1 + W_2)$ = dim. W_1 + dim. W_2 - dim. $(W_1 \cap W_2)$.

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Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Honours), Part-II,

Paper-IV

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.
All questions carry equal marks.

Group-A

1. (a) Solve : $\frac{d^2y}{dx^2}+\cot x\frac{dy}{dx}+(4\operatorname{cosec}^2 x).y=0$.

   (b) Solve the differential equation by the method of variation of parameters. $\frac{d^2y}{dx^2}+a^2y=\operatorname{cosec} ax.$

2. Solve : (a) $p(p+x)=y(x+y)$ (b) $y=(1+p)x+ap^2$.

3. (a) Solve the differential equation $(8p^3-27)x=12p^2y$ and investigate whether a singular solution exists

   (b) Obtain the primitive and singular solution of the equation $xp^2-2yp+4x=0$.

Group-B

4. (a) Prove that $\nabla\cdot(\vec{u}\times\vec{v})=\vec{v}\cdot(\nabla\times\vec{u})-\vec{u}\cdot(\nabla\times\vec{v})$.

   (b) Prove that $\nabla\cdot(\nabla\times\vec{u})=0$ or div. curl $\vec{u}=0$.

5. (a) If $\vec{a}$ and $\vec{b}$ are constant vectors and $\vec{r}=(x,y,z)$, then prove that :

   $$\nabla\cdot\left\{\vec{a}\times\left(\nabla\left(\frac{1}{\vec{r}}\right)\right)\right\}=0.$$

   (b) Find the unit normal vector to the level surface $x^2+y-z=4$ at the point (2, 0, 0).

6. (a) Evaluate : $\frac{d^2}{dt^2}\left\{\left(\vec{r}\times\frac{d\vec{r}}{dt}\right)\times\frac{d^2\vec{r}}{dt^2}\right\}$.

   (b) Prove that $[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}]=[\vec{a}\quad \vec{b}\quad \vec{c}]^2$.

7. (a) Find the volume of the parallelopiped whose edges are represented by :

   $\vec{i}+\vec{j}+\vec{k},\ \vec{i}-\vec{j}+\vec{k},\ \vec{i}+2\vec{j}-\vec{k}$.

   (b) Prove that : $[\vec{a}\ \vec{b}\ \vec{c}][\vec{p}\ \vec{q}\ \vec{r}]=\begin{bmatrix}\vec{a}\cdot\vec{p} & \vec{b}\cdot\vec{p} & \vec{c}\cdot\vec{p}\\ \vec{a}\cdot\vec{q} & \vec{b}\cdot\vec{q} & \vec{c}\cdot\vec{q}\\ \vec{a}\cdot\vec{r} & \vec{b}\cdot\vec{r} & \vec{c}\cdot\vec{r}\end{bmatrix}$.

Group-C

8. State and prove the necessary and sufficient condition for the principle of virtual work.

9. (a) Show that the modulus of an elastic string is equal to the force which would stretch a light string to twice its natural length.

   (b) What are the forces that can be neglected during forming the equation of virtual work.

10. In a simple Harmonic motion if u, v, w be the velocities at distances a, b, c respectively from a fixed point on the straight line which is not the centre of the force, then Show that the periodic time is given by the equation:

$$\frac{4\pi^2}{T}(a-b)(b-c)(c-a)=\begin{vmatrix}u^2 & v^2 & w^2\\ a & b & c\\ 1 & 1 & 1\end{vmatrix}.$$

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Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Subsidiary), Part-II,

Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Eight questions, selecting atleast one from each group. All questions carry equal marks.

Group-A

1. Evaluate any two of the following integrals:

   (a) $\int \frac{2x+3}{\sqrt{x^2+x+1}}dx$ (b) $\int \frac{x^2 dx}{(1-x^4)\sqrt{1+x^4}}$ (c) $\int \frac{dx}{\sqrt{(x-a)(x-b)}}$

2. Evaluate any two of the following:

   (a) $\int_0^\pi \frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$ (b) $\int_0^\pi \frac{x\,dx}{1+\sin x}$ (c) $\int_0^\pi \frac{dx}{a+b\cos x}$

3. (a) Obtain a reduction formula for $\int \sin^m x \cos^n x\,dx$.

   (b) Evaluate $\underset{n\to\infty}{\text{limit}}\left[\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\text{.........}+\frac{n}{n^2+(n-1)^2}\right]$.

4. Find the area of the loop of the curve
   $x^3 + y^3 = 3axy$.

5. Show that the length of the loop of the curve $3ay^2 = x\,(x - a)^2$ is $\frac{4a}{\sqrt{3}}$.

6. Find the area of the surface of revolution formed by revolving the loop of the curve $9ay^2 = x\,(3a - x)^2$ about the x-axis.

7. Find the volume of the solid generated by the revolution of the upper half of the loop of the curve $y^2 = x^2(2 - x)$.

8. Find the perimeter of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$.

9. Solve the following Differential equations:

   (a) $\frac{d^2y}{dx^2} - y = x\sin x$ (b) $\frac{d^2y}{dx^2} + a^2y = \sec ax$.

10. Solve the following Differential equations:

    (a) $y = x\left\{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2\right\}$.

    (b) $p(p + x) = y(x + y)$.

Group-B

11. Find the equation of the sphere which passes through the point (α, β, γ) and the circle
    $x^2 + y^2 + z^2 = a^2, z = 0$.

12. Find the equation of the right circular cylinder which passes through the circle
    $x^2 + y^2 + z^2 = 9, x - y + z = 3$.

13. (a) Prove that a hyper plane is a closed set.

    (b) Define a convex set $S \subseteq R^2$ and prove that the sphere is a convex set.

Group-C

14. If forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x \cos \alpha + y \sin \alpha = p$. Find the magnitude of the resultant and its line of action.

15. Find the equation of line of action of co-planar forces and its resultant.

16. Define simple Harmonic Motion and show that how two simple Harmonic motions can be compounded in a straight line.

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Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Honours), Part-III

Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. Prove that (R^n, d) is complete where d on R^n is defined as $d(x, y) = \left[\sum_{i=1}^{n} |x_i - y_i|^2\right]^{\frac{1}{2}}$.

2. (a) Let (X, d) be a metric space and $A \subseteq X$ then show that A is closed if and only if $A \subseteq \overline{A}$.
   (b) If M and N are two subsets of a metric space (X, d) then show that $\overline{MUN} = \overline{M} \cup \overline{N}$.

3. (a) In a metric space (x,d) prove that the union of an arbitrary collection of open sets is open.
   (b) In a metric space (x,d) prove that any finite intersection of open sets in X is open.

4. (a) State and prove Minkowsky's in equality.
   (b) State and prove Cauchy Schwartz inequality.

5. (a) Define the convergence of a sequare (x_n) in a metric space (x,d) an prove that limit of sequence in (x,d) if it exists is unique.
   (b) Define a Cauchy sequence in a metric space (x,d) and prove that every convergent sequences in (x,d) is a Causchy sequence in (x,d).

Group 'B'

6. (a) Let (X,T_1) and (Y, T_2) be two Topological spaces then a function $f: X \to Y$ is $T_1 \to T_2$ continuous if and only if for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.
   (b) Let (X, T_1) and (Y, T_2) be two Topological spaces then a mapping $f: X \to Y$ is open if and only if $f(A^{\circ}) \subseteq [f(A)]^{\circ}$ for every subset A of X.

7. Let (X, T) be a Topological space and A and B are any two subsets of X and $\overline{A}$ denotes the closure of A then prove that :

(a) $\overline{\phi} = \phi$	(b) $A \subseteq \overline{A}$	(c) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$
(d) $\overline{A \cup B} = \overline{A} \cup \overline{B}$	(e) $(\overline{A \cap B}) \subseteq \overline{A} \cap \overline{B}$	(f) $\overline{\overline{A}} = A$

Group 'C'

8. (a) Prove that every bounded monotonic function $f:[a,b] \to R$ is R-integrable on [a,b].
   (b) If a function f is continuous on [a,b] then prove that it is integrable on [a,b].

9. (a) if f and g are bounded and R-integrable on [a,b] then prove that f+g is also bounded and R-integrable on [a,b] and $\int_a^b \{f(x) + g(x)\}dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.
   (b) if f and g are two bounded and R-integrable functions in [a,b] then prove that fg is bounded and R-integrable in [a,b].

Group 'D'

10. (a) Find the radius of convergence of the series $\sum \frac{n^n x^n}{n}$.
    (b) Prove that the series $\sum\left(\frac{\cos n\theta}{n^2}\right)$ is convergent for all real values of θ.

11. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n \log n (\log \log n)^p}$.

Nalanda Open University
Annual Examination - 2020
B.Sc. Mathematics (Honours), Part-III
Paper-VI

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Sate and prove Caley's Theorem.
   (b) Prove that every group is isomorphic to a group of one-one onto functions.
2. (a) Prove that if a group G has four elements then it must be abelian, group.
   (b) Prove that the order of an element a of a group G is equal to the order of $f(a)$.
3. (a) State and prove Lagrange's Theorem.
   (b) Prove that the order of every element of a finite group is a divisor of the order of the group.

Group 'B'

4. (a) If R is a commutative ring with unity element then show that R is a field if and only if it has non-trivial ideals.
   (b) If $f(x) = x^4 + x^3 - 3x^2 - x + 2$ and $g(x) = x^4 + x^3 - x^2 + x - 2$.
   Then find the g.c.d. of $f(x)$ and $g(x)$ as polynomials over Q.
5. (a) Define a normal subgroup of a group G. Show that every subgroup of an abelian group is normal.
   (b) If f is a homomorphism of a group G into a group G'. Then prove that the Kernel K of G is a normal subgroup of G.

Group 'C'

6. (a) Prove that $N \times N$ is countable.
   (b) If A_i is countably infinite set then prove that $\bigcup_{i=1}^{\infty} A_i$ is countably infinite set.
7. (a) State and prove Zorn's Lemma.
   (b) Prove that $2^{No} = c$.
8. (a) State and prove Schroder-Bernstein Theorem.
   (b) Show that the set of all real numbers in [0, 1] is not denumerable.

Group 'D'

9. (a) Find the domain of the convergence of the series $\sum_{n=1}^{\infty} \frac{1.3.5..........(2n-1)}{n\,!}\left(\frac{1-z}{z}\right)^n$.
   (b) Find the radius of convergence of the series $\frac{z}{2} + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^3 + ...\infty$.
10. State and prove Cauchy integral formula.
11. (a) If $f(z) = u + iv$ is analytic function and $u - v = e^x(\cos y - \sin y)$ find $f(z)$ in terms of z.
    (b) Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. satisfies laplacis equation.

Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Honours), Part-III

Paper-VII

Time: 3.00 Hrs. **Full Marks: 80**

(Graphpaper may be supplied)

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. Solve the L.P.P. problem by simplex method.
   Maximize $z = 4x_1 + 10x_2$. Subject to the conditions.
   $2x_1 + x_2 \leq 50$
   $2x_1 + 5x_2 \leq 100$
   $2x_1 + 3x_2 \leq 90$
   $x_1, x_2 \geq 0$.

2. Maximize $z = 3x + 5y + 4z$. Subject to the conditions.
   $2x + 3y \leq 8$
   $2y + 5z \leq 10$
   $3x + 2y + 4z \leq 15$
   $x, y, z \geq 0$.

3. (a) Prove that the set of all feasible solutions of a linear programming problem constitutes a convex set.
   (b) Prove that a sphere is a convex set.

Group 'B'

4. (a) Solve $\frac{dx}{dt} + 4x + 3y = t^2$ and $\frac{dy}{dt} + 2x + 5y = e^{2t}$.
   (b) Solve $t\frac{dx}{dt} + y = 0$ and $t\frac{dy}{dt} + x = 0$.

5. (a) Solve $(2xz - yz)dx + (2yz - zx)dy - (x^2 - xy + z^2)dz = 0$.
   (b) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$.

6. (a) Solve $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$
   (b) Solve $(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$.

7. (a) Solve $r = a^3t$ by Monge's method.
   (b) Solve $r - t\cos^2 x + p\tan x = 0$ by Monge's method.

8. (a) Solve $(p^2 + q^2)y = qz$ by Charpits method.
   (b) Solve $pxy + pq + qy - yz = 0$ by Charpit's methods.

Group 'C'

9. Find the centre of pressure of a vertical circle of radius 'a' wholly immersed in a homogeneous liquid with its centre at a depth h below the free surface.

10. (a) Find the potential of a circular disc at a point distant h on the axis from the centre.
    (b) Find the attraction of a circular disc at an external point at height h.

Nalanda Open University

Annual Examination - 2020

B.Sc. Mathematics (Honours), Part-III

Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Five questions. All questions carry equal marks.

1. (a) Express the following functions and their differences in the factorial notation.

   (i) $y = x^4 - 12x^3 + 42x^2 - 30x + 9.$ (ii) $y = 2x^3 - 3x^2 + 3x - 10$

   (b) If $f(x)$ and $g(x)$ are any functions of x then prove that :

   (i) $\Delta\left[f(x)\, g(x)\right] = f(x)\Delta\, g(x) + \Delta g(x+1) f(x) \;=\; f(x+1)\Delta\, g(x) + g(x)\,\Delta f(x)$

   (ii) $\Delta\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)\, g(x+1)}.$

2. (a) Show that if n is a positive integer then : $(x\Delta)^n \cup_x = (x + n - 1)^{(n)}\, \Delta^n \cup_x$.

   (b) Prove that : $\cup_1\, x + \cup_2\, x^2 + \cup_3\, x^3 + \ldots\ldots\ldots\ldots$

   $= \dfrac{x}{1-x} \cup_1 + \dfrac{x^2}{(1-x)^2} \Delta \cup_1 + \dfrac{x^3}{(1-x)^3} \Delta^2 \cup_1 + \ldots\ldots\ldots\ldots$

3. (a) Prove that : $\cup_x - \cup_{x+1} + \cup_{x+2} - \cup_{x+3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   $= \dfrac{1}{2}\left[\cup_{x-\frac{1}{2}} - \dfrac{1}{2}\Delta^2 \cup_{x-\frac{3}{2}} + \dfrac{1.3}{2!8^2}\Delta^4{}_{x-\frac{5}{2}} + \ldots\ldots\ldots\right].$

   (b) Show that if Δ operates on n, then :

   $\Delta\dbinom{n}{x+1} = \dbinom{n}{x}$ and hence deduce that $\displaystyle\sum_{n=1}^{N}\dbinom{n}{x} = \dbinom{N+1}{x+1} - \dbinom{1}{x+1}.$

4. (a) Prove that : $\Delta^n O^{n+1} = \dfrac{n(n+1)}{2}\Delta^n O^n.$

   (b) Prove that : $\dfrac{\Delta^n O^m}{\lfloor\underline{n}} = \dfrac{n\Delta^n O^{m-1}}{\lfloor\underline{n}} + \dfrac{\Delta^{n-1} O^{m-1}}{\lfloor\underline{n-1}}.$

5. (a) Find the sixth term of the series : 8 + 12 + 19 + 29 + 42 + ...

   (b) Estimate the missing figure in the following table :

x :	1	2	3	4	5
$f(x)$:	2	5	7	X	32

6. (b) Find the polynomial of the lowest degree which assumes the values 3, 12, 15, –21. When x has the values 3, 2, 1, –1 respectively.

   (a) What is the form of the function of the following table.

x :	0	1	4	5
$f(x)$:	8	11	68	123

7. Find the maximum and minimum values of the function tabulated below.

x :	0	1	2	3	4	5
$f(x)$:	0	0.25	0	2.25	16.00	56.25

8. If $f(20) = 14, f(24) = 32, f(28) = 35, f(32) = 40.$

   Then by Gauss's forward formula show that $f(25) = 33 \cdot 49$.

9. (a) Find the solution of the difference equation

   $u_{x+4} - 7u_{x+1} + 12u_x = \cos x.$

   (b) Evaluate $\displaystyle\int_0^6 \frac{1}{1+x^2}dx$ using simpson's $\dfrac{1}{3}$ rd rule.

10. Solve the equation $2 + \log_{10}^{x} = 2e^{-x}$ by the method of iteration.