# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
## PAPER–I
(Advanced Abstract Algebra)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. (a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
   (b) State and prove Remainder theorem.
2. (a) Prove that if F is a field and K is an extension of F then an element $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.
   (b) Prove that every finite extension of a field F is algebraic.
3. (a) Find necessary and sufficient condition on a, b so that the splitting field of irreducible polynomial $x^3 + ax + b$ has degree 3 over $Q$.
   (b) Show that the splitting field of $x^4 + 1 = 0$ is $Q(\sqrt{2}, i)$ whose degree over $Q$ is 4.
4. If $\Psi$ be an isomorphism of a field $F_1$ onto a field $F_2$ such that $\alpha \Psi = \alpha'$ for every $\alpha \in F_1$ then prove that there is an isomorphism $\phi$ of $F_1[x]$ on to $F_2[t]$ with the property $\alpha \phi = \alpha \Psi = \alpha'$ for each $\alpha \in F_1$.
5. If F is a field of characteristics 0 and a, b are algebraic over F then prove that there exists an element $c \in F[a, b]$ such that $F[a, b] = F[c]$ i.e. $F[a, b]$ is a simple extension.
6. (a) If K be the splitting field of $x^n - a \in F[x]$. Then show that $G(K, F)$ is a solvable group.
   (b) Show that the Galois group of $x^4 + x^2 + 1$ is the same as that of $x^6 - 1$.
7. Let F be a field of characteristics O. Then prove that a polynomial $f(x) \in F(x)$ is solvable by radicals over F if and only if its splitting field K over F has solvable Galois group $G[K, F]$.
8. (a) Prove that the group $G[Q(\alpha), Q]$ where $\alpha^5 - 1$ and $\alpha \neq 1$, is isomorphic to cyclic group of order 4.
   (b) Show that every element in a finite field can be written as the sum of two squares.
9. (a) Prove that every finitely generated module is homomorphic image of a finitely generated free module.
   (b) Prove that every homomorphic image of a Noetherian (artinian) module is Noetherian (artinian).
10. (a) State and prove Schur's theorem.
    (b) Prove that the necessary and sufficient condition for a module M to be a direct sum of its two sub modules $M_1$ and $M_2$ are that (i) $M = M_1 + M_2$, (ii) $M_1 \cap M_2 = \{0\}$.

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**EXAMINATION PROGRAMME-2020**
**M.Sc. Mathematics, Part-I**

| Date | Papers | Time | Examination Centre |
|---|---|---|---|
| 05.04.2021 | Paper–I | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |
| 07.04.2021 | Paper–II | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |
| 09.04.2021 | Paper–III | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |
| 12.04.2021 | Paper–IV | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |
| 15.04.2021 | Paper–V | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |
| 17.04.2021 | Paper–VI | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |
| 22.04.2021 | Paper–VII | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |
| 26.04.2021 | Paper–VIII | 2.30 PM to 5.30 PM | A. N. College, Boring Road, Patna-800013 |

# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
## PAPER–II
(Real Analysis)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. State and prove Implicit function theorem.

2. (a) Define a function of bounded variation clearly and prove that a bounded monotonic function is a function of bounded variation.

   (b) If $f \in BV[a, b]$ and $c \in [a, b]$ then prove that $f \in BV[a, c]$ and $f \in BV[c, b]$ and conversely moreover $V(f, a, b) = V(f, a, c) + V(f, c, b)$.

3. Let *f* be bounded and *g* a non-decreasing function on [a, b]. Then prove that $f \in RS(g)$ if and only if for every $\in > 0$ there exists a partition P of [a, b] such that
$U(P, f, g) - L(P, f, g) < \in$

4. Let $f_1, f_2 \in RS(g)$ on [a, b] then prove that $f_1 + f_2 \in RS(g)$ on [a, b] and
$\int_a^b (f_1 + f_2) dg = \int_a^b f_1 \, dg + \int_a^b f_2 \, dg$.

5. If *f* is defined by $f(x, y) = \dfrac{x^2 y^2}{x^2 + y^2}, \; x^2 + y^2 \neq (0, 0)$

   $= 0$ otherwise.

   Then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$ although neither $f_{xy}$ nor $f_{yx}$ is continuous at $(0, 0)$. Account for the equality.

6. Discuss the continuity and differentiability of the function *f* defined by $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ when $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ at $(0, 0)$.

7. State and prove Schwarz's theorem for a function of two variables.

8. If $u_1, u_2, u_3, \ldots\ldots\ldots, u_n$ are functions of $y_1, y_2, \ldots\ldots., y_n$ and $y_1, y_2, y_3 \ldots\ldots., y_n$ are functions of $x_1, x_2, x_3 \ldots\ldots., x_n$ then show that
$$\frac{\partial(u_1, u_2, \ldots.., u_n)}{\partial(x_1, x_2, \ldots.., x_n)} = \frac{\partial(u_1, u_2, \ldots.., u_n)}{\partial(y_1, y_2, \ldots.., y_n)} \cdot \frac{\partial(y_1, y_2, \ldots.., y_n)}{\partial(x_1, x_2, \ldots.., x_n)}.$$

9. (a) Show that the sequence $(f_n)$ where $f_n(x) = \dfrac{x}{1 + nx^2}$ converges uniformly on R.

   (b) Prove that the series $\sum u_n(x) \, v_n(x)$ will be uniformly convergent on [a, b] if

   (i) $(v_n(x))$ is a positive monotonic decreasing sequence converging uniformly to zero for $a \leq x \leq b$.

   (ii) $|v_n(x)| = \left| \sum_{v=1}^{n} u_r(x) \right| < k$ for every value of *x* in [a, b] and for all integral value of *n* where *k* is a fixed number independent of *x*.

10. Find the radius of convergence of the following power series.

    (i) $\displaystyle\sum_{n=1}^{\infty} \frac{\lfloor n}{n^n} z^n$ (ii) $\displaystyle\sum_{n=0}^{\infty} \frac{(\lfloor n)^2}{(2n)!} z^n$

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
## PAPER–III
(Measure Theory)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. (a) If A, B are L-measurable sets then show that (i) $A \cup B$ (ii) $A \cap B$ are L-measurable.
   (b) Show that the measure of an enumerable set is Zero.
2. (a) Prove that a necessary and sufficient condition for a Set $S \subseteq R^k$ to be L-measurable is that for every Set $W \subseteq R^k$
   $$|W| = |W \cap S| + |W \cap S^c|$$
   i.e. $m(W) = m(W \cap S) + m(W \cap S^c)$
   (b) If $A_1, A_2, A_3, \ldots\ldots\ldots\ldots A_n$ are L-measurable mutually disjoint sets in $R^k$ then show that
   (i) $W \subseteq R^k \Rightarrow \left|W \cap \sum_{r=1}^{n} A_r\right| = \sum_{r=1}^{n} |W \cap A_r|$
   (ii) $m\left(\sum_{r=1}^{n} A_r\right) = \sum_{r=1}^{n} m(A_r)$
3. Let $(A_r)$ be a sequence of L-measurable sets such that
   (i) $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots\ldots\ldots$ and $A = \bigcap_{r=1}^{\infty} A_r$
   (ii) $m(A_1) < \infty$
   then $m(A) = \underset{r \to \infty}{lt}\ m(A_r)$
4. (a) Prove that a necessary and sufficient condition for a function $f$ to be L-measurable is that it is the limit of a convergent sequence of simple functions.
   (b) Prove that the class M of L-measurable functions is closed with respect to all arithmetical operations.
5. State and prove bounded convergence theorem.
6. (a) If f is L-integrable over X then prove that $\left|\int_X f d\mu\right| \le \int_X |f| d\mu$.
   (b) If E and F are measurable sets and f is a integrable function on E + F then show that $\int_{E+F} f d\mu = \int_E f d\mu + \int_F f d\mu$
7. (a) State and prove Lebesgue dominated convergence theorem.
   (b) Use Lebesgue dominated convergence theorem to evaluate $\lim_{n \to \infty} \int_0^1 f_n(x) dx$ where $f_n(x) = \frac{n^{1/2} x}{1 + n^2 x^2}$.
8. Use bounded convergence theorem for the function $f_n(x) = \frac{nx}{1 + n^2 x^2}$ to show that whether bounded convergence theorem is true or not in [0, 1].
9. Prove that every absolutely continuous function is an indefinite integral of its own derivative.
10. Let x be a Lebesgue point of a function f(t) then show that the indefinite integral $F(x) = f(a) + \int_a^x f(t) dt$ is differentiable at each point x and $F'(x) = f(x)$.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
### PAPER–IV
(Topology)
***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. (a) What do you mean by $T_2$-space or a Hausdorff space. Prove that every discrete space is a Hausdorff space.
   (b) Define the closure of a Set $A \subseteq X$ where $(X, T)$ is a topological space. If $(X, T)$ is a topological space and A and B are any two subsets of X then prove that,
   (i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (ii) $\overline{A \cap B} = \overline{A} \cap \overline{B}$ (iii) $\overline{\overline{A}} = \overline{A}$

2. (a) Define a Topological space, Indiscrete Topology, Discrete topology, co-finite topology and co-countable topology.
   (b) Let $\{T_i : i \in I\}$ where I is an arbitrary set, be a collection of topologies for X. Then show that the intersection $\cap\{T_i : i \in I\}$ is also a topology for X.

3. Define a metrizable space and Equivalent metrics giving one suitable example for each.

4. (a) Let $(X, T_1)$, $(Y, T_2)$ be two topological spaces. Then prove that a mapping $f : X \to Y$ is closed if and only if $f(\overline{A}) = \overline{f(A)}\ \forall\ A \subseteq X$.
   (b) Let $(X, T_1)$, $(Y, T_2)$ be two topological spaces. Then prove that a mapping $f : X \to Y$ is $T_1 - T_2$ continuous if and only if for every subset A of X, $f(\overline{A}) = \overline{f(A)}$.

5. (a) What do you mean by Hereditary property of a topological space $(X, T)$. Let $(Y, T_Y)$ be a sub space of $(X, T)$ and let $\mathrm{B}$ be a base for T. Then show that $B_Y = \{B \cap Y : B \in \mathrm{B}\}$ is a base for $T_Y$.
   (b) Define a subspace of a topological space $(X, T)$. Let $(X, T)$ be a topological space and $Y \subseteq X$. Then show that the collection $T_y = \{G \cap Y : G \in T\}$ is a topology on Y.

6. Prove that the union of any family of connected sets having a non-empty intersection is connected.

7. If $(X, T_1)$, $(Y, T_2)$ are two topological spaces and let $f : X \to Y$ be one-one, onto. Then prove that f is homeomorphism iff $f(\overline{A}) = \overline{f(A)}\ \forall\ A \subseteq X$.

8. (a) Prove that every second countable space is separable.
   (b) Prove that a topological space (X, T) is a $T_1$-space iff every singleton subset {x} of X is a T-closed set.

9. (a) Prove that every convergent sequence in a Hausdorff space has unique limit.
   (b) Prove that every closed-subspace of a normal space is normal.

10. (b) Prove that every compact sub space of a Hausdorff space is closed.
    (a) Prove that every closed subset of a compact space is compact.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
## PAPER–V
(Linear Algebra, Lattice Theory and Boolean Algebra)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

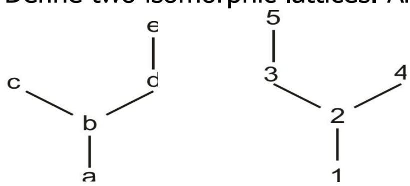
***Answer any Five Questions.***
***All questions carry equal marks.***

1. Let $T : U \to V$ be a linear transformation. Then prove that $\dim \cdot \ker(T) + \dim \cdot range(T) = \dim \cdot domain(T)$

2. Prove that a linear transformation E on a linear subspace L is a projection on some subspace if and only if it is idempotent i.e. $E^2 = E$.

3. (a) Define a Lattice and dual of a statement in a Lattice. Give two examples to make it clear.
   (b) Define a sub lattices with two examples.

4. (a) Define two isomorphic lattices. Are the two lattices shown in the figure isomorphic ?




   (b) Define a bounded lattice and show that every finite lattice is bounded.

5. (a) What do you mean by a complemented lattice. Prove that if L be a bounded distributive lattice then complements are unique if they exist.
   (b) If L is a complemented lattice with unique complements then the join irreducible elements of L other than O are its atom prove it.

6. (a) Consider the Boolean algebra $D_{210}$ the divisors of 210. Find the number of sub algebras of $D_{210}$.
   (b) Find the maxiterms of the Boolean algebra $P(A)$ consisting of the subsets of $A = \{a, b, c\}$.

7. (a) Express $E = Z(x' + y) + y'$ in complete sum of products form.
   (b) Write the Boolean expression $E(x, y, z)$ first as a sum of products and then in complete sum of products form.

8. (a) Prove that the row space and the column space of a matrix A have the same dimension.
   (b) Find the basis for the row space of the following matrix A and determine its rank $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & 1 & 5 \end{bmatrix}$.

9. (a) Prove that all bases for a vector space V have the same number of vectors.
   (b) Show that the set $\{x^2 + 1, 3x - 1, -4x + 1\}$ is linearly independent and the set $\{x + 1, x - 1, -x + 5\}$ is linearly dependent.

10. (a) If $W_1$ and $W_2$ are finite dimensional sub spaces of a vector space V, then prove that $W_1 + W_2$ is also finite dimensional and
    $\dim \cdot W_1 + \dim \cdot W_2 = \dim \cdot (W_1 \cap W_2) + \dim \cdot (W_1 + W_2)$.
    (b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for $R^3$ and express each of the standard basis vectors as linear combinations of $\alpha_1, \alpha_2$ *and* $\alpha_3$.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
## PAPER–VI
(Complex Analysis)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. (a) Find the bilinear transformation which maps the points $z = -2, 0, 2$ into points $w = 0, i, -i$ respectively.

   (b) Show that the transformation $w = \dfrac{5 - 4z}{4z - 2}$ transforms the circle $|z| = 1$ into a circle of radius unity in w-plane and find the centre of the circle.

2. (a) Derive necessary and sufficient condition for $f(z)$ to be analytic in polar co-ordinates.

   (b) If $f(z)$ is an analytic function of z, prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) |R\, f(z)|^2 = 2|f'(z)|^2$.

3. (a) Examine the behaviour of the power series $\sum_{n=2}^{\infty} \dfrac{z^n}{n(\log n)^2}$ on the circle of convergence.

   (b) Find the domain of convergence of the series $\sum_{n=1}^{\infty} \dfrac{1 \cdot 3 \cdot 5 .........(2n-1)}{n!} \left(\dfrac{1-z}{2}\right)^n$.

4. State and prove poison's integral formula.

5. State and prove Laurent' theorem.

6. State and prove Cauchy integral formula i.e. If $f(z)$ is analytic within and on a closed contour C and if a is any point within C then $f(a) = \dfrac{1}{2\pi i} \int_C \dfrac{f(z)}{z - a} \cdot dz$.

7. State and prove maximum modulus principle.

8. Find the Laurent's series of the function $f(z) = \dfrac{1}{z^2(1 - z)}$ about $z = 0$ and expand $\dfrac{1}{z^2 - 3z + 2}$ for (i) $0 < |z| < 1$ (ii) $1 < |z| < 2$ (iii) $(z) > 2$.

9. Evaluate any two of the following integrals :–

   (a) $\int_0^{\pi} \dfrac{a\, d\theta}{a^2 + Sin^2 \theta}, \; a > 0$

   (b) $\int_0^{\pi} \dfrac{Cos\, 2\theta}{5 + 4\, Cos\theta} do$

   (c) $\int_0^{\infty} \dfrac{Cos\, mx}{x^2 + a^2} dx$

   (d) $\int_0^{\infty} \dfrac{\log(1 + x^2)}{1 + x^2} dx$

10. (a) Discuss the nature of singularities of the following functions

    (i) $\dfrac{1}{z(z-1)^2}$ (ii) $\dfrac{z}{1 + z^4}$ (iii) $\dfrac{1}{z(e^z - 1)}$

    (b) Evaluate the residue of $\dfrac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
### PAPER–VII
(Theory of Differential Equations)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. State and prove Cauchy-Peano existence theorem.
2. State and prove Ascolli's Lemma.
3. Show that the function given below satisfy Lipschitz condition in the rectangle indicated and hence find Lipschitz constant $f(x, y) = (y + y^2)\frac{Cos\, x}{2}$, $|y| \leq 1, |x - 1| \leq ½$.
4. Define a linear system and show that it satisfies Lipschitz condition and the set of solutions form a vector space.
5. Compute the first three successive approximations for the solution of the Initial Value problem $y^1 = y^2, \quad y(0) = 1$.
6. Test the stability of the non-linear system $\frac{dx}{dt} = x + 4y - x^2, \frac{dy}{dt} = 6x - y + 2xy$. Further make a comment on stability.
7. Find the nature of the critical point (0, 0) of the system of equations $\frac{dx}{dt} = 3x + 4y$ and $\frac{dy}{dt} = 3x + 2y$.
8. Find the series solution of Bessel's differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0$.
9. (a) Define fundamental matrix and show that a necessary and sufficient condition that a solution matrix G to be fundamental matrix is $G(x) \neq 0$ for $x \in I$.

   (b) Solve the differential equations by matrix method

   $$\frac{dx_1}{dt} = 9x_1 - 8x_2$$

   $$\frac{dx_2}{dt} = 24x_1 - 9x_2$$

   The initial conditions for which are $x_1(0) = 1, x_2(0) = 0$.
10. (a) Describe orthogonal property of Laguerre polynomial.

    (b) What is the meaning of generating function for Legendre polynomial ? Hence find it.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-I
## PAPER–VIII
(Set Theory, Graph Theory, Number Theory and Differential Geometry)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. (a) Prove that the set of all real numbers R is uncountable.
   (b) If A and B are countable sets then show that A × B is also countable.
2. (a) Prove that Zorn's lemma implies well ordering theorem.
   (b) What is Axiom of choice ? Show that the Axiom of choice is equivalent to Zermelo's postulates.
3. (a) Prove that every set can be well ordered.
   (b) If $\alpha, \beta, \lambda$ are any three Cardinal numbers then prove that
   (i) $\alpha + (\beta + \lambda) = (\alpha + \beta) + \lambda$ (ii) $\alpha(\beta + \lambda) = \alpha\beta + \alpha\lambda$
4. (a) Find g.c.d. of 28 and 49 and express it as a linear combination of 28 and 49.
   (b) State and prove division algorithm in theory of numbers.
5. (a) Show that $T(n) = \left[\frac{n}{1}\right] + \left[\frac{n}{2}\right] + \left[\frac{n}{3}\right] + \ldots\ldots\ldots + \left[\frac{n}{n}\right]$.
   (b) Solve the congruence $x^3 \equiv 5$ (mod 13).
6. (a) Factorize 493 by Euler's Factorization method.
   (b) If $a_n$ is the nth term of the Fibonaced sequence and $\alpha = \frac{1 + \sqrt{5}}{2}$ then $a_n > \alpha^{n-1}\ \forall\ n > 1$.
7. Define an Umbilic. Prove that in general three lines of curvature pass through an umbilic.
8. Show that for a geodesic

   $T^2 = (K - K_1)(K - K_2)$ where

   K is curvature and T is torsion.
9. (a) If a tree has n vertices of degree 1, two vertices of degree 2, four vertices of degree 4 then find the value of n.
   (b) Show that a complete graph of n vertices is a planner if $n \le 6$.
10. (a) If a tree has n vertices of degree 4 then find the value of n.
    (b) Prove that an undirected graph is a tree iff there is a unique path between any two vertices.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-II
## PAPER–IX
(Numerical Analysis)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions. All questions carry equal marks.***
***Calculator is Allowed.***

1. (a) Express $2x^3 - 3x^2 + 3x - 10$ and its difference in factorial notation, the interval of differencing being unity.
   (b) Show that $\Delta^n x^{(n)} = \underline{|n}\, h^n$ and $\Delta^{n+1} x^{(n)} = 0$.
2. Prove that the divided differences can be expressed as the product of multiple integrals.
3. (a) Obtain the estimate of the missing numbers in the following table.

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| f(x) | 1 | 8 | ? | 64 | ? | 216 | 343 | 512 |

   (b) Use the method of separation of symbols to prove that $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \ldots\ldots\ldots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n-1}$.
4. Find $f'(7.50)$ from the following table.

| x | 7.47 | 7.48 | 7.49 | 7.50 | 7.51 | 7.52 | 7.53 |
|---|---|---|---|---|---|---|---|
| y = f(x) | .193 | .195 | .198 | .201 | .203 | .206 | .208 |

5. Find the polynomial of fifth degree from the following data $u_0 = -18,\ u_1 = 0,\ u_3 = 0,\ u_5 = -248,\ u_6 = 0,\ u_9 = 13140$.
6. (a) Solve the equation $y_{h+2} - 4y_{h+1} + 4yh = 0$.
   (b) Find the sum to n terms of the series whose $x^{\text{th}}$ term is $2^x\left(x^3 + x\right)$.
7. (a) Find the root of the equation $x^3 - 6x - 11 = 0$ which lies between 3 and 4.
   (b) By using the method of iteration find a real root of $2x - \log_{10}^{x} = 7$.
8. Find the value of $\int_0^1 \frac{dx}{1 + x^2}$ by using Simpson's $\frac{1}{3}^{\text{rd}}$ and $\frac{3}{8}^{\text{th}}$ rule, hence obtain the approximate value of $\pi$ in each case.
9. (a) Fit a second degree parabola to the following :–

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 1 | 1.8 | 1.3 | 2.5 | 6.3 |

   (b) Obtain an approximation in the sense of the principle of least squares in the form of polynomial of second degree to the function $f(x) = \frac{1}{1 + x^2}$ in the range $-1 \le x \le 1$.
10. (a) Solve the equation $3x - Cosx - 1 = 0$ by false position method and Newton Raphson method.
   (b) Use synthetic division to solve $f(x) = x^2 - 1.0001x + 0\cdot9999 = 0$ in the neighbourhood of x = 1.

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**EXAMINATION PROGRAMME-2020**

## M.Sc. Mathematics, Part-II

| Date | Papers | Time | Examination Centre |
|---|---|---|---|
| 28.01.2021 | Paper–IX | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |
| 30.01.2021 | Paper–X | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |
| 02.02.2021 | Paper–XI | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |
| 04.02.2021 | Paper–XII | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |
| 06.02.2021 | Paper–XIII | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |
| 09.02.2021 | Paper–XIV | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |
| 11.02.2021 | Paper–XV | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |
| 13.02.2021 | Paper–XVI | 2.30 PM to 5.30 PM | Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna |

# NALANDA OPEN UNIVERSITY

## M.Sc. Mathematics, Part-II

## PAPER–X

(Functional Analysis)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions. All questions carry equal marks.***
***Calculator is Allowed.***

1. State and prove Riesz Lemma.

2. State and prove closed graph theorem.

3. (a) Show that the inner product space is jointly continuous.

   (b) If x and y are any two vectors in an inner product space then prove that $|(x, y)| \leq \| x \| \cdot \| y \|$.

4. Consider a real number p such that $1 \leq p < \infty$. Denote $l_p$ the space of all sequences $x = (x_1, x_2, x_3, \ldots\ldots\ldots\ldots)$ of scalars such that $\sum_{n=1}^{\infty} | x_n | p < \infty$ with norm defined by $\| x \|_p = \left( \sum_{n=1}^{\infty} | x_n | p \right)^{1/p}$. Show that $l_p$ is a Banach space.

5. (a) If x and y are any two vectors on a Hilbert space H then show that $\| x - y \|^2 + \| x + y \|^2 = 2\left( \| x \|^2 + \| y \|^2 \right)$.

   (b) Show that a normed linear space is a metric space under the property $\big| \| x \| - \| y \| \big| \leq \| x \| - \| y \|$.

6. If $f$ and $g$ belong to $L^p(a, b)$ where p > 1 then prove that $\| f + g \|_p \leq \| f \|_p + \| g \|_p$.

7. If N and N' be normal linear spaces and let T : N $\rightarrow$ N' be any linear map. If N is finite dimensional then prove that T is continuous or bounded.

8. State and prove open mapping theorem.

9. (a) If T is a normal operator on a Hilbert space H then prove that $\| T^2 \| = \| T \|^2$.

   (b) Prove that if M and N are closed linear sub spaces of a Hilbert space H $s \cdot t \;\; M \perp N$, then M + N is also closed.

10. If N and N' be normal linear spaces and T be a linear transformation of N into N'. Then show that the following conditions are equivalent to one another.

    (i) T is continuous.

    (ii) T is continuous at the origin (i.e. $x_n \rightarrow 0 \Rightarrow T(x_n) \rightarrow 0$)

    (iii) T is bounded (i.e. $\exists$ a real number $K \geq 0 \;\; s \cdot t \;\; \| T(x) \| \leq K \| x \| \; \forall \; x \in N$).

    (iv) If $S = \{ x : \| x \| = 1 \}$ is closed sphere in N then the image T(S) is a bounded set in N'.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-II
## PAPER–XI
(Partial Differential Equations)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions. All questions carry equal marks.***

1. Solve
   (a) $\left(D^2 + 2DD' + D'^2\right)z = e^{3x + 2y}$
   (b) $\left(D^2 + 3DD' + 2D'^2\right)z = x + y$

2. (a) Solve $p^2 x + q^2 y = z$ by Jaicobi's method.
   (b) Find the complete integral of $\left(p^2 + q^2\right)x = pz$. (use Charpit's method)

3. (a) Solve $(q + 1)s = (p + 1)t$ by Monge's method.
   (b) Solve $pt - qs = q^3$, by Monge's method.

4. Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its general solution.

5. Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

6. (a) Find the characteristics curve of $2yu_x + \left(2x + y^2\right)u_y = 0$ passing through (0, 0).
   (b) Find the integral surface of the equation $(2xy - 1)p + \left(z - 2x^2\right)q = 2(x - yz)$ which passes through the line $x = 1,\ y = 0,\ z = 1$.

7. Show that the general solution of the wave equation $c^2\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial^2 u}{\partial t^2}$ is $u(x, t) = \phi(x + ct) + \psi(x - ct)$ where $\phi$ and $\psi$ are arbitrary functions.

8. Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\left(\frac{\partial u}{\partial t}\right)$ satisfying the conditions $0 = u(0, t) = u(l, t),\ u(x, 0) = \left(x - x^2\right)$.

9. Transform the Laplaces equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ in the cylindrical co-ordinates.

10. Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(x, 0) = u(x, b) = 0$ for $0 < x < a$, and $u(a, y) = f(y)$ for $0 < y < 6$.

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# NALANDA OPEN UNIVERSITY

## M.Sc. Mathematics, Part-II

### PAPER–XII

(Analytical Dynamics)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. State and prove Jaicobi-Poisson theorem.

2. (a) A particle moves in a plane under a central force depending on its distance from the origin. Then construct the Hamiltonian of the system and derive Hamilton's equation of motion.
   (b) Explain the principle of least action and hence establish it in terms of arc length of a particle path.

3. (a) Derive the formula for the Kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of Kinetic energy.
   (b) Prove that in a simple dynamic system $T + V =$ constant where T and V have their usual meaning.

4. Determine the kinetic energy and the moment of momentum of a rigid body rotating about a fixed axis.

5. (a) A particle of mass m moves in a force field whose potential in spherical co-ordinates is given by $V = \frac{\lambda \, Cos\, \theta}{r^2}$, then write down the Hamilton Jaicobi equation and derive the complete solution.
   (b) State and prove Jaicobi-theorem.

6. Derive Euler's equation of motion for the motion of rigid body about a fixed point.

7. (a) Derive Lagrange's equation of impulsive motion in a Holonomic dynamical system.
   (b) A bead is sliding on a uniformly rotating wire in a force free space. Derive its equation of motion.

8. (a) Define the generating function of a transformation and give an example of a generating function of transformation.
   (b) Show that the transformation $Q = q \tan p$, $P = \log(Sin\, p)$ is canonical.

9. (a) Explain the terms (i) degree of freedom, (ii) Constraints, (iii) generalized co-ordinates and classify the dynamical systems based on different types of constraints.
   (b) Explain the difference between possible displacement and virtual displacement. Give one example of each.

10. (a) Use Routhian equation of motion to determine the motion of a uniform heavy rod turning about one end which is fixed.
    (b) Construct Routhian function and Routh's equation for the solution of a problem involving cyclic and non-cyclic co-ordinates.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-II
## PAPER–XIII
(Fluid Mechanics)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. State and prove Kelvin's circulation theorem.
2. Derive Cauchy-Riemann differential equation in polar form.
3. Derive Euler's equation of Fluid motion.
4. Obtain the equation of continuity in spherical polar co-ordinates.
5. (a) Show that the two dimensional irrotational motion, stream function satisfies Laplace's equation.
   (b) A two dimensional flow field in given by $\psi = xy$, then (i) show that the flow is irrotational (ii) Find the velocity potential (iii) find the stream lines and potential lines.
6. (a) What do you mean by source and sink ? Find the complex potential due to a source of strength m placed at the origin.
   (b) Show that $u = Axy$, $v = A(a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Determine the stream function of fluid motion.
7. Derive the equation of energy for an incompressible fluid motion with constant fluid properties.
8. Obtain the boundary layer equations in two dimensional flow.
9. (a) What type of motion do the following velocity components constitute ?
   $u = a + by - cz$, $v = d - bx + ez$, $w = f + cx - ey$ where a, b, c, d, e, f are arbitrary constants.
   (b) Find the principal stresses and principal directions of stress at a point (1, 1, 1) if the components of the stess tensor are given by

$$\sigma y = \begin{pmatrix} 0 & 2y_3 & 0 \\ 2y_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

10. Show that the vorticity vector $\Omega$ of an incompressible viscous fluid moving under no external forces satisfies the differential equation.
    $\frac{D\Omega}{Dt} = (\Omega \bullet \nabla) q + v \ \nabla^2 \ \Omega$ Where $v$ is the kinematic of viscosity.

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-II
## PAPER–XIV
(Operation Research)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions. All questions carry equal marks.***

1. (a) Find basic feasible solution of the system $2x_1 + x_2 + 4x_3 = 11, \quad 3x_1 + x_2 + 5x_3 = 14$.
   (b) Reduce feasible solution $x_1 = 2, x_2 = 4, x_3 = 1$ of the system $2x_1 - x_2 + 2x_3 = 2$ and $x_1 + 4x_2 = 18$ to a basic feasible solution and mention its kind (degenerate or non-degenerate).

2. Using simplex method solve L.P.P.
   Max $z = 4x_1 + 10x_2$
   Subject to the condition $2x_1 + x_2 \le 50,\ 2x_1 + 5x_2 \le 10,\ 2x_1 + 3x_2 \le 90;\ x_1 \ge 0,\ x_2 \ge 0$.

3. (a) Obtain the feasible solution of non-linear programming
   Min $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
   Subject to $x_2 \le 8, \quad x_1 + x_2 \le 10, \quad x_1 \ge 0,\ x_2 \ge 0$.
   (b) By using Lagrange's multiplier method solve the NLPP
   $z = a\,x_1^2 + b\,x_2^2 + c\,x_3^2$ where $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 1$.

4. Explain in details the dual simplex method elaborating each step.

5. Apply two phase method to compute the solution of
   Min $z = x_1 + x_2$
   Subject to $2x_1 + x_2 \ge 4, \quad x_1 + 7x_2 \ge 7; \qquad x_1 \ge 0, \quad x_2 \ge 0$.

6. (a) Describe the method of constructing the solution of 'Game Problem' where the game is without saddle point.
   (b) Solve the game problem whose pay off matrix is $\begin{bmatrix} 6 & 2 & 7 \\ 1 & 9 & 3 \end{bmatrix}$.

7. Find the dual of the following L.P.P.
   Min $z = x_1 + x_2 + x_3$
   S.t. $x_1 - 3x_2 + 4x_3 = 5, \quad x_1 - 2x_2 \le 3, \quad 2x_2 - x_3 \ge 4; \quad x_1, x_2 \ge 0;$ $x_3$ is unrestricted in sign.

8. (a) Define hyper plane and hyper sphere. Prove that every hyper plane in $R^n$ is a convex set.
   (b) Prove that the intersection of any finite number of convex sets is a convex set.

9. Solve the following transportation problem.

| | To | | | Supply |
|---|---|---|---|---|
| | 1 | 2 | 3 | |
| From 1 | 2 | 7 | 4 | 5 |
| From 2 | 3 | 3 | 1 | 8 |
| From 3 | 5 | 4 | 7 | 7 |
| From 4 | 1 | 6 | 2 | 14 |
| Demand | 7 | 9 | 18 | 34 |

10. Solve the following assignment problem.

| Task \ Man | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 1 | 3 | 2 | 3 | 6 |
| B | 2 | 4 | 3 | 1 | 5 |
| C | 5 | 6 | 3 | 4 | 6 |
| D | 3 | 1 | 4 | 2 | 2 |
| E | 1 | 5 | 6 | 5 | 4 |

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-II
## PAPER–XV
(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. Find the Laplace transform of
   (a) $L\left\{Sin\sqrt{t}\right\}$ (b) $L\left\{Sin^2 at\right\}$ (c) $L\left\{e^{at}\ Cos\,bt\right\}$ (d) $L\left\{e^{-t}\,Cos^2 t\right\}$

2. Find (a) $L^{-1}\left\{\dfrac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$ (b) $L^{-1}\left\{\dfrac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right\}$

3. Using Laplace transform solve the following differential equations
   (a) $(D+1)^2 y = t$, given that $y=-3$, when t = 0 and y = -1 when t = 1.
   (b) Solve $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}-3y=2\,Sin\,t$, given that $y=\dfrac{dy}{dt}=0$ when t = 0.

4. (a) Find the Fourier cosine transform of $e^{-x^2}$.
   (b) Find the Fourier sine transform of $x\,e^{-\frac{x^2}{2}}$ .

5. (a) State and prove convolution theorem of Fourier transforms.
   (b) Find Fourier transform of $F(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$ and hence prove that $\int_0^{\infty}\dfrac{Sin^2 ax}{x^2}dx=\dfrac{\pi a}{2}$.

6. (a) Explain about the Fredholm integral equations of three kinds.
   (b) Describe Firedholm integral equation and voltera integral equation.

7. The maintenance and re-sale value per year of a machine whose purchase prices is Rs. 7000/- is given below :—

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Maintenance Cost in Rs. | 900 | 1200 | 1600 | 2100 | 2800 | 3700 | 4700 | 5900 |
| Re-sale Value in Rs. | 4000 | 2000 | 1200 | 600 | 50 | 400 | 400 | 400 |

   When should the Machine be replaced.

8. (a) What do you mean by Christoffel's symbols and prove that
   $[ij,\,k]+[jk,\,i]=\dfrac{\partial g_{ik}}{\partial x^j}=\partial_j g_{ik}$.
   (b) Show that $\left\{\begin{matrix} i \\ i\ \ j\end{matrix}\right\}=\dfrac{\partial\log\left(\sqrt{g}\right)}{\partial x^j}=\dfrac{\partial\left(\log\sqrt{-g}\right)}{\partial x^j}$.

9. (a) Define the inner and outer product of two tensors and prove that the outer product of two tensors is a tensor of rank equal to the sum of ranks of the two tensors.
   (b) Prove that the inner product of tensors $A^i_{ij}$ and $B^{ij}_i$ is a tensor of rank three.

10. (a) Prove that a skew symmetric tensor of rank two has $\dfrac{N}{2}(N-1)$ independent components.
    (b) Show that any linear combination of tensors of type (r, s) is a tensor of type (r, s).

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# NALANDA OPEN UNIVERSITY
## M.Sc. Mathematics, Part-II
## PAPER–XVI
(Programming in 'C')

***Annual Examination, 2020***

**Time : 3 Hours.** **Full Marks : 80**

***Answer any Five Questions.***
***All questions carry equal marks.***

1. What is an Operator ? Describe different types of operators in C with examples.
2. What is control statement is C Language ? Explain with the help of an example.
3. Discuss integer constant, floating point constant and character constant. What are the rules for constructing integer contents ?
4. What is function ? Are functions required when writing a C program ?
5. Write a programme in C to find the roots of a quadratic equation.
6. What are reserved words ? What is the difference between the expression "++a" and "a++" ? Explain with examples.
7. What are logical errors and how does it differ from syntax errors ? Write a program in C to swap the value of two variables.
8. What is the purpose of the switch statement ? How does switch statement differ from the other statements ?
9. What is debugging ? When is the "void" keyword used in a function ?
10. Write short notes on any two of the following :—
    (i) GOTO Statement
    (ii) Variables
    (iii) Branching statements
    (iv) Recursion

* * *

### M.Sc. Mathematics, Part–II, Paper–XVI
### Practical Counselling & Examination Programme, 2020

| अनुक्रमांक | परामर्श कक्षा की तिथि | परामर्श कक्षा का समय | परीक्षा की तिथि | परीक्षा का समय |
|---|---|---|---|---|
| 180290001 to 180290190 | 01.03.2021 to 03.03.2021 | 11.00 AM to 2.00 PM | 04.03.2021 | 11.30 AM to 1.30 PM |
| 180290191 to 180290395 | | 2.00 PM to 5.00 PM | 04.03.2021 | 2.30 PM to 4.30 PM |
| 180290396 to 180290650 | 05.03.2021 to 08.03.2021 | 11.00 AM to 2.00 PM | 09.03.2021 | 11.30 AM to 1.30 PM |
| 180290651 to 180290665 & All Old Students | | 2.00 PM to 5.00 PM | 09.03.2021 | 2.30 PM to 4.30 PM |
| **Venue : School of Computer Education & IT, Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001** | | | | |

अतः विद्यार्थियों को हिदायत दी जाती है कि वे अपना Admit Card अवश्य लेकर आयें अन्यथा परामर्श कक्षाओं एवं परीक्षाओं से वंचित हो सकते हैं ।

इस कार्यक्रम में किसी भी परिस्थिति में परिवर्तन नहीं होगा ।