Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Honours), Part-I
Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. State and prove fundamental theorem of equivalence relation.
2. What do you mean by a partial order relation and total order relation and well ordered set. Give one example of each.
3. If A, B, C, D are any three non-empty sets then prove that.
   (a) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D) \cup (A \times D) \cup (B \times C)$
   (b) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
4. (a) Prove that an infinite union of denumerable sets is denumerable.
   (b) Define a Lattice, complete Lattice and set an example of a Lattice which is not a complete Lattice.
5. (a) If $f : X \to Y$ and $A \subseteq Y$, $B \subseteq Y$ then show that
   $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ and $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$
   (b) Define an equivalence relation and equivalence classes of sets giving one example of each.

Group - B

6. (a) Prove that a group G is abelian if $b^{-1}a^{-1}ba = e$, $\forall$ a, b $\in$ G.
   (b) If H_1, H_2 are subgroups of a group G then show that $H_1 \cap H_2$ is also a subgroup of G.
7. (a) Prove that if a group G has four elements then it must be abelian.
   (b) Prove that the order of every element of a finite group is a divisor of the order of the group.
8. (a) Define a group and show that the four fourth roots namely 1, –1, i, –i form a group with respect to multiplication.
   (b) Prove that G = {0, 1, 2, 3, 4, 5} is a finite abelian group of order 6 with respect to addition modulo 6.

Group - C

9. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
10. (a) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$.
    (b) If A and B are two non-singular matrices of the same order then prove that $(AB)^{-1} = B^{-1}A^{-1}$.
11. (a) Solve the following system of linear equations by matrix method.
    $x + y + z = 6$, $2x + y - 3z = -5$, $3x - 2y + z = 2$
    (b) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ then find the value of $A^2 - 4A + 3I$.
12. (a) State and prove De-Moiver's theorem.
    (b) Find the condition so that the equation $x^4 - px^3 - qx^2 + rx + s = 0$ may have its roots in arithmetical progression.

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Honours), Part-I
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) If y = $e^{a\sin^{-1}x}$ then prove that
$(1-x^2 y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$
(b) If $y = (x^2 - 1)^n$ then prove that
$(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$

2. (a) State and prove Taylor's theorem.
(b) Find the Lagrange's form of remainder after n terms in the expansion of $e^{ax}\cos bx$ in powers of x.

3. (a) Evaluate $\underset{x\to 0}{lt}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. (b) Evaluate $\underset{x\to 0}{lt}\frac{xe^x - \log(1+x)}{x^2}$

4. (a) If the normal at any point to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with x-axis then show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$.
(b) If $u = \log(x^2 + y^2 + z^2 - 3xyz)$ the show that :
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{3}{(x^2+y^2+z^2)^2}$$

5. (a) Prove that the radius of curvature for the pedal curve $p = f(r)$ is given by $P = r\frac{dr}{dp}$.
(b) Find the asymptote to the curve : $(x^2 + y^2)(x + 2y + 2) = x + 9y + 2$

Group - B

6. Evaluate any two of the following:
(a) $\int \frac{x^2}{x^4+1}dx$ (b) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ (c) $\int \frac{dx}{x^3+a^3}$

7. (a) Obtain the reduction formula for $\int \cos^m x \sin nx\, dx$.
(b) Evaluate $\underset{n\to\infty}{lt}\left(\sum_{n=1}^{n-1}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}\right)$

8. Evaluate the following integrals.
(a) $\int_0^{\pi/2}\log(\sin x)dx$ (b) $\int_0^{\pi/2}\frac{xdx}{a^2\sin^2 x + b^2\cos^2 x}$

9. Find the surface of the solid obtained by revolving the curve $r^2 = a^2 \cos 20$ about the initial axis.

10. Find the area of the loop of the curve $(x + a)^2(x + 2a) + y^2x = 0$

Group – C

11. (a) Find the polar equation of the conic in the form $\frac{l}{r} = 1 + e\cos\theta$.
(b) Find the polar equation of the tangent at any point of it to the conic $\frac{l}{r} = 1 + e\cos\theta$.

12. (a) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y - 15 = 0$.
(b) If the tangent to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts on the co-ordinate axis a, b, c, respectively then show that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{r^2}$.

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Subsidiary), Part-I
Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

Answer any ***Five*** *questions, selecting atleast* ***One*** *question from each group.*
All questions carry equals marks.

Group - A

1. If A, B, C are any three non-empty sets then prove that
   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

2. (a) Let $f : X \to Y$, $A \subseteq Y$, $B \subseteq Y$ then show that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
   (b) What do you mean by an equivalence relation. Give tow examples of it.

3. (a) For a finite group G, prove that order of every element of G is finite and less than or equal to the order of G.
   (b) Let $G = \{1, w, w^2\}$ where w is an imaginary cube roof of unity then G is a group with respect to multiplication.

4. (a) What do you mean by an abelian group? If a group G has four elements then prove that it must be abelian.

5. (a) Let f be a homomorphism of a group G onto a group G' with Kernel, $K = \{x \in G : f(x) = e')$ where e' is the identity element of G' then show that K is a normal sub group of G.
   (b) Let f be a homomorphism of a group G into a group G' then prove that.
   (i) f(e) = e' where e is the identity of G and e' that of G'.
   (ii) $f(a^{-1}) = \{f(a)\}^{-1} \ \forall \ a \in G$.
   (iii) If the order of $a \in G$ is finite then order of f(a) is the divisor of the order of a.

Group - B

6. (a) State and prove De-Moivre's theorem.
   (b) Decompose $\log(\alpha + i\beta)$ into real and imaginary parts.

7. (a) Test the convergence of the series.
   $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots\ldots\ldots\ldots \infty$.
   (b) Test the convergence of the series whose n^{th} terms is $\left(\sqrt{n^2+1} - \sqrt{n^2-1}\right)$.

8. (a) State and prove Cauchy general principle of convergence of a real sequence.
   (b) Show that the sequence (a_n) where $a_n = \sqrt{n^2+4n} - n$ is convergent.

Group - C

9. Deduce the polar equation of the conic in the form $\frac{l}{r} = 1 + e\cos\theta$.

10. (a) State and prove Euler's theorem on Homogeneous functions of two variables.
    (b) If f(x, y) = x cos y + y cos x then prove that :
    $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

11. Find the condition under which a general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse.

12. (a) Prove that: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.
    (b) Prove that: $[\vec{a} \times \vec{b} \ \ \vec{b} \times \vec{c} \ \ \vec{c} \times \vec{a}] = 2[\vec{a} \ \vec{b} \ \vec{c}]^2$.

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Honours), Part-II, Paper-III

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.
All questions carry equal marks.

Group-A

1. (a) State and prove fundamental theorem of classical analysis.
   (b) State and prove theorem of least upper bound.
2. (a) State and prove theorem of greatest lower bound.
   (b) Show that any nonempty open set is union of open internals.
3. (a) Define a closed set. Prove that the intersection of any number of closed sets is closed.
   (b) Prove that between two distinct real numbers there lie infinity of irrationals and rationals.

Group-B

4. (a) Define a convergent sequence and show that it is bounded.
   (b) Show that a bounded monotonic increasing sequence tends to its least upper bound.
5. (a) Show that the sequence (a_n) defined by $a_1 = \sqrt{7}, a_{n+1} = \sqrt{7 + a_n}$ converges to a positive roof of the equation $x^2 - x - 7 = 0$.
   (b) Let $x_1 = 1, x_2 = \sqrt{2 + x_1}, x_3 = \sqrt{2 + x_2}, \ldots\ldots, x_{n+1} = \sqrt{2 + x_n}$. Show that the sequence (x_n) is convergent and the limit of convergence is 2.
6. (a) State and prove Cauchy's n^{th} root test for convergence of an infinite series.
   (b) Test the convergence of the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots\ldots\ldots\ldots \infty$.
7. (a) Test the convergence of the series whose n^{th} term is $\sqrt{n^2+1} - \sqrt{n^2-1}$.
   (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{1+n^2}, \forall\, x > 0$.
8. (a) State and prove Raabe's test.
   (b) Test for the convergence of the series $\sum \frac{(n+1)(n+2)}{(n+3)(n+4)}$.

Group-C

9. (a) Let V be a vector space and W_1, W_2 are finite dimensional subspaces of V. Then show that $W_1 + W_2$ is finite dimensional and
   dim $.W_1$ + dim . W_2 = dim $(W_1 \cap W_2)$ + dim $(W_1 + W_2)$.
   (b) Prove that any two bases of a finite dimensional vector space have the same number of elements.
10. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$.
    (b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$.

******

Examination Programme, 2019

(Bachelor of Science (Part-II)

All Science Subjects Except B.Sc Geography & Home Science (Honours)

(बी.एस.सी भूगोल और गृह विज्ञान (ऑनर्स) को छोड़कर विज्ञान के सभी आनर्स विषय)

Date	Paper	Time	Name of Examination Centre
28/5/2019	HONOURS PAPER – III	**12.00 to 3.00 pm**	Nalanda Open University, Patna
30/5/2019	HONOURS PAPER – IV	**12.00 to 3.00 pm**	Nalanda Open University, Patna
01/6/2019	Hindi 100 or Ur 50+Hn 50	**12.00 to 3.00 pm**	Nalanda Open University, Patna
03/6/2019	(SUB.) (Botany- II)	8.00 to 11.00 am	Nalanda Open University, Patna
04/6/2019	(SUB.) (Mathematics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
06/6/2019	(SUB.) (Chemistry - II)	8.00 to 11.00 am	Nalanda Open University, Patna
07/6/2019	(SUB.) (Physics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
08/6/2019	(SUB.) (Zoology - II)	8.00 to 11.00 am	Nalanda Open University, Patna
11/6/2019	(SUB.) (Geography - II)	8.00 to 11.00 am	Nalanda Open University, Patna
13/6/2019	(SUB.) (Home Science- II)	8.00 to 11.00 am	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2019

B.Sc. Mathematics (Honours), Part-II,

Paper-IV

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.
All questions carry equal marks.

Group-A

1. Solve any two of the following differential equations:

   (a) $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6 = 0$ (b) $(px - y)(x - py) = 2p$. (c) $(x - a)p^2 + (x - y)p - y = 0$

2. (a) Find the orthogonal Trajectory of the family of cardoids $r = a(1 + \cos\theta)$

   (b) Prove that the system of confocal conic

   $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal.

3. (a) Solve : $\frac{d^2y}{dx^2} + a^2 y = \sec ax$ by using variation of parameter.

   (b) Solve $\frac{d^2y}{dx^2} + \frac{1}{x}.\frac{dy}{dx} + 4x^2 y = x^4$ use method of change of variable.

Group-B

4. (a) Show that $\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right] = \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$.

   (b) Prove that : $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$.

5. (a) Prove that $\nabla\times(\vec{u}\pm\vec{v}) = \nabla\times\vec{u} \pm \nabla\times\vec{v}$.

   (b) Prove that : $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot(\nabla\times\vec{u}) - \vec{u}\cdot(\nabla\times\vec{v})$

6. (a) Prove the $\frac{d}{dt}(\vec{u}\times\vec{v}) = \vec{u}\times\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\times\vec{v}$.

   (b) Prove the $\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \vec{u}\cdot\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\cdot\vec{v}$.

7. State and prove the necessary and sufficient condition of the principle of virtual work.

Group-C

8. State and prove the necessary and sufficient condition for equilibrium of a system of co-planar forces also find the equation of line of action of the resultant.

9. Derive the tangential and normal velocities and accelerations in polar co-ordinates.

10. Define simple Harmonic motion. If in a simple harmonic motion *u*, *v*, *w* be the velocities at distances *a*, *b*, *c* from a fixed point on the straight line which is not the centre of the force. Show that the periodic time T is given by the equation:

$$4\pi^2(a-b)(b-c)(c-a) = T\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$

******

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Subsidiary), Part-II,
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Eight questions, selecting atleast one from each group. All questions carry equal marks.

Group-A

1. Evaluate any two of the following integrals:

(a) $\int \frac{dx}{\sin x(3+2\cos x)}$ (b) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (c) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$

2. Evaluate any two of the following:

(a) $\int_0^{\pi/4} \log(\tan x)dx$ (b) $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}dx$ (c) $\int_0^a \frac{\log(1+x^2)}{1+x^2}dx$

3. Find the reduction formula for:

(a) $\int_0^{\pi/2} \sin^m x\cos^n x\,dx$ (b) $\int \sin^m x\cdot\cos nx\,dx.$

4. Find the perimeter of the loop of the curve
$9ay^2 = (x-2a)(x-5a)^2$.

5. Find the area between the curve $y^2(a+x) = (a-x)^2$ and its asymptote.

6. (a) Evaluate $\underset{n\to\infty}{lt}\left[\frac{1^2}{1^3+n^3}+\frac{2^2}{2^3+n^3}+..........+\frac{n^2}{n^3+n^3}\right]$.

(b) Evaluate $\underset{n\to\infty}{lt}\frac{[(n+1)(n+2)(n+3)\,...........(n+n)]}{n}$

7. Find the volume of revolution of the loop of the curve $y^2(a+x) = x^2(a-x)$ about the x-axis.

8. Solve the following differential equations:

(a) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{2x}$ (b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}+4y=x^2.$

9. Solve:

(a) $y = 2px + p^2$ (b) $y = px = x^4p^2$.

Group-B

10. (a) Prove that the intersection of a finite number of convex sets is convex set.
(b) Define a convex set and a hyper plane and prove that a hyperplane is a convex set.

11. Find the volume of a Tetrahedron, the co-ordinates of whose vertices are given.

12. (a) Find the equation of the sphere which passes through the point (α, β, γ) and the circle $x^2 + y^2 + z^2 = a^2$, $z = 0$.

(b) Find the equation of the right circular cylinder whose axis is given by $\frac{x}{1}=\frac{y}{0}=\frac{z}{2}.$ and radius $\sqrt{7}$.

Group-C

13. State and prove principle of virtual work.

14. What do you mean by Simple Harmonic Motion, derive an expression for time period.

15. Deduce general conditions for equilibrium of a system of co-planar forces.

16. (a) State and establish the principle of energy.
(b) Analyze the motion of a body under inverse square law.

******

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Honours), Part-III
Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1.(a) Prove that in a metric space (x, d) each open sphere is an open set.

(b) Let (X, d) be a metric space. Show that a function $d^* : X \times X \to R$ difined by $d^* = \frac{d(x,y)}{1+d(x,y)}$ is also a metric for X.

2. State and prove Minkowsky's inequality.

3. (a) If $1 < p < \infty$, $1 < q < \infty$ such that $\frac{1}{p}+\frac{1}{q}=1$ and a, b are real numbers such that $a > 0$, $b > 0$ then prove that $ab \le \frac{a^p}{p}+\frac{b^q}{q}$.

4. Prove that every metric space in first countable.

5. Show that every metric space is T_2 – space.

Group 'B'

6. Let (X, T) is a topological space and A and B are subsets of X. If $\overline{A}$ denotes the closure of A then show that :

(a) $(\overline{A \cap B}) = \overline{A} \cap \overline{B}$ (b) $(\overline{A \cup B}) = \overline{A} \cup \overline{B}$ (c) $\overline{\overline{A}} = \overline{A}$

7. What do you mean by a Hausdorff space, Show that every discrete topological space is a Hausdorff space.

Group 'C'

8. Prove that if a bounded function f is R-integrable over $[a, b]$ and M and m are bounds of f then $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$ *if* $b \ge a$.

9. State and prove Darboux's theorem.

10. State and prove necessary and sufficient condition for R-integrability of a bounded function f over [a, b].

Group 'D'

11. Discuss the convergence of the following series :

(a) $1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\frac{1}{5^p}+\frac{1}{6^p}+...........\infty$.

(b) $\sum_{n=2}^{\infty}\frac{1}{n\log n(\log\log n)^p}$

12. Show that the sum of the series $1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+........$ is half the sum of the series $1-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}+\frac{1}{7}..............$.

Examination Programme-2019
B.Sc (Part–III) Mathematics Honours

Date	Papers	Time	Examination Centre
09/4/2019	Honours Paper–V	8 to 11 AM	Nalanda Open University, Patna
12/4/2019	Honours Paper–VI	8 to 11 AM	Nalanda Open University, Patna
13/4/2019	Honours Paper–VII	8 to 11 AM	Nalanda Open University, Patna
15/4/2019	Honours Paper–VIII	8 to 11 AM	Nalanda Open University, Patna
17/4/2019	Paper –XV (General Studies)	8 to 11 AM	Nalanda Open University, Patna

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Honours), Part-III
Paper-VI

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. Show that any ring can be embedded in a ring with unity.
2. Define a ring homomorphism. If $f: R \to R'$ be a homomorphism of a ring R onto a ring R' then show that f is a homomorphism iff Keruel f $f = \{0\}$
3. Define a principal ideal ring and show that the ring of integers is a principal ideal ring.
4. Show that the union of two ideals is again an ideal.
5. Prove that the set of all polynomials in $Z[x]$ with constant term O is prime ideal in $Z[x]$.
6. (a) Define an automorphism of a group G. Let $x \in G$, then prove that the function f defined by $f(g) = x^{-1}gx$ for $g \in G$ is an auto morphism of G.

   (b) If G is a group, then for every element $g \in G$, prove that $C_a(g)$ is a Subgroup of G.

Group 'B'

7. State and prove Cantor's Theorem.
8. (a) Prove that $2^{No} = c$, where symbols have their usual meaning.

   (b) For cardinal numbers α, β, γ prove that

   (i) $\alpha^{\beta} . \alpha^{\gamma} = \alpha^{\beta + \gamma}$ (i) $(\alpha . \beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$ (iii) $(\alpha^{\beta})^{\gamma} = \alpha^{\beta\gamma}$.
9. (a) Introduce the concept of order types and construct the product of two order types.

   (b) If X is any non-empty set then show that card $(P(x))$ is 2 where $P(x)$ the power set of X.

Group 'C'

10. Obtain the necessary and sufficient condition for differentiability of a complex valued function.
11. State and prove Cauchey integral formula.
12. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchey-Riemann differential equations are satisfied.

    (b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^2} dz$. Where C is the circle $|z| = 3$.

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Honours), Part-III
Paper-VII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Define a convex set, the subset of R^n and show that the finite intersection of convex sets is a convex set.
   (b) Prove that every hyperplane is convex.
2. Use simplex method to solve :
   Maximize : $z = 3x_1 + 9x_2$
   Subject to $x_1 + 4x_2 \leq 8$, $x_1 + 2x_2 \leq 4$, and $x_1 \geq 0$, $x_2 \geq 0$.
3. (a) Prove that the set of all feasible solutions of a linear programming problem constitutes a convex set.
   (b) Define convex combination of vectors in R^n. Prove that the set of convex combinations of a finite number of linearly independent vectors $v_1, v_2, v_3, \ldots\ldots\ldots, v_n$ is a convex set.

Group 'B'

4. (a) Solve $\dfrac{dx}{x^2 - y^2 - z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$.
   (b) Solve $\dfrac{dx}{dt} + 4x + 3y = l$ and $\dfrac{dy}{dt} + 2x + 5y = e^t$.
5. Solve by using Charpit's method $(p^2 + q^2)x = pz$.
6. Solve :
   (a) $pz - qz = z^2(x + y)^2$.
   (b) $(y + z)p + (z + x)q = x + y$.
7. Test for integrability and hence solve the equation
   $(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$
8. Use Monge's method to find the complete solution of the equation
   $2x^2r - 5xys + 2y^2t + 2(px + qy) = 0$

Group 'C'

9. State and prove Laplace theorem in cartesian form.
10. Find the attraction of a uniform sphere at an external point of it.

Nalanda Open University
Annual Examination - 2019
B.Sc. Mathematics (Honours), Part-III
Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Five questions. All questions carry equal marks.

1. Use Gauss-Jordan method to solve the system of equations

$$x_1 + 2x_2 + x_3 = 8,\ 2x_1 + 3x_2 + 4x_3 = 20 \text{ and } 4x_1 + 3x_2 + 3x_3 = 16,$$

taking initial condition $x_1 = 0, x_2 = 0, x_3 = 0$.

2. (a) Derive simpson's $\frac{3}{8}$th rule for numerical integration.

   (b) Use weddl's rule to evaluate $\int_0^{10} \frac{1}{x+1} dx$.

3. Appliying analytical method for finding roots of an equation based on Rolle's theorem and demonstrate on $3x - \sqrt{1+\sin x} = 0$.

4. (a) Discuss Newton-Raphson's method to obtain approximate value of root of $f(x) = 0$.

   (b) By using synthetic division solve $f(x) = x^3 - x^2 - (1.001)x + 0.9999 = 0$ in the neighbourhood of $x = 1$

5. Describe Newton-Gregory formula for backward interpolation.

6. (a) Explain the meaning of the operators E and Δ. and show that E and Δ are commutative with respect to variables.

   (b) Evaluate $\Delta^3(1 - x)(1 - 2x)(1 - 3x)$ and $\Delta^n(e^{ax+b})$ where a and b are constants.

7. (a) Describe Pieard's method of successive approximation.

   (b) Apply Runge Kutta mehtod for the solution of first order differential equation.

8. (a) Explain Gauss's method of elimination for the solution of a system of a system of m equations in m variables.

   (b) Solve the following system of equations

$$x_1 + \frac{1}{2}\cdot x_2 + \frac{1}{3}\cdot x_3 = 1$$

$$\frac{1}{2}\cdot x_1 + \frac{1}{3}\cdot x_2 + \frac{1}{4}\cdot x_3 = 0$$

$$\frac{1}{3}\cdot x_1 + \frac{1}{4}\cdot x_2 + \frac{1}{5}\cdot x_3 = 0$$

9. (a) Derive Trapezoidal and simpson's one third rule to numerical integration.

   (b) Solve difference equation

$$\cup_{x+1} = 2^x \cup_x .$$

10. (a) State and prove Adam's predictor formula.

    (b) Describe Milne corrector formula.