NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–I
(Advanced Abstract Algebra)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove fundamental theorem of Galois theory.
2. State and prove Jordan-Holder theorem on any group.
3. Define Homomorphism and Kernel of homomorphism from a module M into a module N. If f is a module homomorphism then f is an isomorphism if and only if $K(f) = 0$. Prove this.
4. What do you mean by extension of a field. Establish the transitivity property of finite extension of a field.
5. State and prove Kronecker's theorem.
6. (a) Prove that in every principal ideal domain, each pair of elements has a greatest common divisor.
   (b) Prove that the range of homomorphism of a module is a sub-module of the module.
7. (a) Define algebraic and simple extension of a field and give an example of each one.
   (b) If a and b are algebraic over a field F then prove that $a + b$, ab, $ab^{-1}(b \neq 0)$ are also algebraic over F.
8. (a) Show that a module M is the direct sum of two modules M_1 and M_2 if and only if (i) $M_1 + M_2$ and (ii) $M_1 \cap M_2 = \{0\}$ are sub modules.
   (b) Define a sub-module of a module M. Show that arbitrary intersection of sub-modules of a module M is a sub-module of M.
9. (a) Prove that if $K = \phi(\sqrt{2})$ where ϕ is the field of all rational numbers then ϕ is the fixed field under the group of automorphism of K.
   (b) Find the Galois group of the equation $x^3 - 2 = 0$ over the field Q of rational numbers.
10. (a) Define a subnormal series of a group. Hence or otherwise form a subnormal series of the additive group of integers.
    (b) Construct all the composition series of Z_{60}.

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Examination Programme, 2019

M.Sc. Mathematics, Part-I

Date	*Papers*	*Time*	*Examination Centre*
15.07.2019	Paper–I	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
17.07.2019	Paper–II	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
19.07.2019	Paper–III	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
23.07.2019	Paper–IV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
25.07.2019	Paper–V	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
27.07.2019	Paper–VI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
29.07.2019	Paper–VII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
31.07.2019	Paper–VIII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–II
(Real Analysis)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove a necessary and sufficient condition for a function f to be R-integrable over [a, b].

   (b) If $f, g \in R(\alpha)$ on [a, b] then prove that $f+g \in R(\alpha)$ and $\int_a^b (f+g)\,d\alpha = \int_a^b f\,d\alpha + \int_a^b g\,d\alpha$.

2. (a) If $f \in R(\alpha)$ and α is monotonically increasing on [a, b], then show that $|f| \in R(\alpha)$ on *[a, b]* and $\left|\int_a^b f\,d\alpha\right| \le \int_a^b |f|\,d\alpha$.

   (b) If $f \in R(\alpha)$ on [a, b] and $\int_a^b f\,d\alpha = 0$ for every f which is monotonic on [a, b] then prove that α must be constant on [a, b].

3. (a) State and prove Bolzano-Weierstrass theorem and give a suitable example of it.

   (b) Deduce Bolzano-Weierstrass theorem from Heine-Borel theorem.

4. Prove that a necessary and sufficient condition for a function f on [a, b] to be of bounded variation is that it can be written as the difference of two monotonically increasing functions on [a, b].

5. (a) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{\lfloor n}{n^n} x^n$.

   (b) State and prove Abel's theorem.

6. If $f : R^2 \to R$ be defined by $f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}$, $(x, y) \ne (0, 0)$ and $f(0, 0) = 0$ then show that $D_{1,2}\, f(0, 0) \ne D_{2,1}\, f(0, 0)$.

7. State and prove inverse function theorem.

8. State and prove implicit function theorem.

9. Find $\frac{\partial(y_1, y_2, y_3, ---, y_n)}{\partial(x_1, x_2, x_3, ---, x_n)}$ where

   $y_1 = x_1(1 - x_2)$

   $y_2 = x_1 x_2 (1 - x_3)$

   $y_3 = x_1 x_2 x_3 (1 - x_4)$

   $y_{n-1} = x_1 x_2 x_3 --- x_{n-1}(1 - x_n)$

   $y_n = x_1 x_2 x_3 --- x_n$

10. What do you mean by the Extreme values of a function in the case of a function of n = variables and find these values in the case of the function defined by $f(x, y, z) = 2xyz - 4zx - 2yz + x^2 + y^2 + z^2 - 2x - 4y + 4z$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–III
(Measure Theory)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If A, B are L-measurable subsets of R^k then prove that $A \cup B$, $A \cap B$ are also L-measurable subsets of R^k .
   (b) Show that the measure of a Denumerable set is Zero.

2. (a) If (A_n) is sequence of L-measurable subsets of R^k such that $A_1 \supseteq A_2 \supseteq A_3 \supseteq$ .... $\supseteq A_n \supseteq A_{n+1}$ .... and $A = \bigcap_{n=1}^{\infty} A_n$ and $m(A_1) < \infty$ then show that A is L-measurable and $m(A) = \underset{n \to \infty}{lt}\, m(A_n)$.
   (b) If (S_r) is a Sequence of L-measurable subsets of R^k then show that $\bigcup_{r=1}^{\infty} S_r$ is also L-measurable.

3. (a) If f is a measurable function then show that $|f|$ is also a measurable function.
   (b) Show that the class of all measurable functions is closed with respect to all algebraic operations.

4. If (f_n) is a sequence of measurable functions then show that the class of all measurable functions is closed with all analytic operations.

5. Give the analytic description of Cantor's Ternary set and show that it is an uncountable set of measure Zero.

6. Define the Lebesque integral of function in details. If f and g are L-integrable then show that $\int (f+g)d\mu \leq \int f\, d\mu + \int g\, d\mu$.

7. State and prove Fatou's Lemma.

8. State and prove Lebesque monotone convergence theorem.

9. State and prove dominated convergence theorem.

10. (a) Examine the L-integrability of $f(x) = \left(x^2 \sin \frac{1}{x^2}\right)$ over [0, 1].
    (b) Verify bounded convergence theorem for $f_n(x) = \frac{nx}{1+n^2x^2}$ $(0 \leq x \leq 1)$, $n = 1, 2, 3, --$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–IV
(Topology)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that every metric space is a normal space.
   (b) Prove that a topological space (X, T) is normal space if and only if each neighbourhood of a closed set *F* contains the closure of some neighbourhoods of *F*.

2. (a) Define hereditary and topological properties and show that the property of a T_1-space is both hereditary and topological.
   (b) Prove that a topological space (X, T) is T_0 -Space if and only if $x, y \in X$ and $x \neq y \Rightarrow \{x\} \neq \{y\}$.

3. (a) What do you mean by a regular space. Prove that a compact Hausdorff space is regular.
   (b) Prove that every compact subspace of the real line is closed and bounded.

4. (a) Give an example of topological space which is a T_1 -space but not a T_2 -space.
   (b) Prove that a finite sub-set of T_1 -space has no cluster point.

5. If X and Y are topological spaces, then prove that X × Y is connected *iff* X and Y are connected.

6. Prove that an arbitrary intersection of topological spaces is a topological space.

7. (a) Define T_3 -space and T_4 -space and prove that every T_4 -space is a T_3 -space.
   (b) Prove that every compact subspace of a Hausdorff space is closed.

8. (a) Prove that in a Hausdorff space every convergent sequence has a unique limit.
   (b) Let (X, T) be a topological space and $A \subseteq X$. Then show that
   (i) $(Int\ A)' = \bar{A}'$, (ii) $(\bar{A})' = Int(A')$

9. (a) Show that connectedness is not hereditary property.
   (b) Introduce the concept of connected and disconnected spaces and show that a topological space X is connected *iff* ϕ and X are its only subsets which are both open and closed.

10. (a) Show that the open interval (0, 1) on the real line R is not compact.
    (b) If (X, T) be a topological space and $A \subseteq X$, $B \subseteq X$ then show that $Int(A \cap B) = Int(A) \cap Int(B)$

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–V

(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Let *V(F)* be a finite dimensional vector space and *W* is a subspace of *V*, then show that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

2. Prove that a linear operator *E* is a projection on some subspace *iff* it is an idempotent.

3. (a) Prove that two real quadratic forms are equivalent iff they have the same rank and index.
   (b) If *f* is a linear functional on a vector space *V(K)* then show that (i) $f(0) = 0$ and $f(-x) = -f(x)$.

4. (a) Prove that a partially ordered set $(P(X), \subseteq)$ is a lattice.
   (b) If *R* is a ring and *L* is a lattice of all ideals of *R*, then prove that *L* is a modular.

5. (a) Prove that a necessary and sufficient condition for a one to one and onto mapping *f* between two lattices to be isomorphism is that f and f^{-1} are both order preserving.
   (b) Define isomorphism between two lattices. Give one example.

6. (a) Show that the relation precedes $(x \leq y)$ in a Boolean algebra *B* is a partial order relation.
   (b) If *B* is a Boolean algebra then prove that for $\forall\, x, y \in B$ the following are equivalent.
   (i) $x \wedge y' = 0$ (ii) $x \vee y = y$ (iii) $x' \vee y = 1$ (iv) $x \wedge y = x$

7. (a) Prove that a Boolean Algebra *B* is a complemented distributive lattice.
   (b) Prove that in Boolean Algebra, the complement of an element is unique.

8. Define a linear transformation and its null space. If *U(f)* and *V(f)* are two vector spaces and *T* is a linear transformation from *U* into *V*, then show that the kernel *T* or null space of *T* is a subspace of *U*.

9. Convert $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$ to Jordan canonical form.

10. (a) Show that the matrix, $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent of index 3.
    (b) Let {(1, 1, 1), (0, 1, 1), (0, 0, 1)} be a basis of Euclidean space R^3, then find its orthonormal basis.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VI
(Complex Analysis)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Find the necessary and sufficient condition for analyticity of a function $f(z)$.
   (b) Show that $u = ½\log(x^2 + y^2)$ is a harmonic function. Also find the analytic function $f(z)$ whose real part is u.

2. (a) State and prove Cauchy-Hadmard theorem for power series.
   (b) Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{2^n + 1}$.

3. Find Taylor's expansion of the function $f(z) = \frac{z}{z^2 + 9}$ around $z = 0$.

4. (a) Describe different kinds of singularities.
   (b) What is the pole of a function ? Also introduce the residue at simple pole and pole of order m.

5. (a) State and prove the necessary and sufficient condition for the transformation $w = f(z)$ to be conformal.
   (b) Show that the transformation $w = \frac{5 - 4z}{4z - 2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the w–plane and hence find its centre.

6. By introducing Bilinear transformation, derive the existence of fixed points of a Bilinear transformation.

7. State and prove Poisson's integral formula.

8. State and prove Cauchy's theorem.

9. Evaluate the following integrals :—
   (a) $\int_0^{2\pi} \frac{d\theta}{1 + a\ Cos\theta}$ *where* $a^2 < 1$
   (b) $\int_0^{\infty} \frac{dx}{(1 + x^2)^2}$

10. Using Cauchy's integral formula evaluate $\int_C \frac{z\,dz}{(9 - z^2)(z + 1)}$, where C is the circle described anticlockwise and having equation $|z| = 2$.

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OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VII
(Theory of Differential Equations)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove Picard's-Lindelof theorem.

2. Define Lipschitz condition in a region. Show that the following function do not satisfy the Lipschitz condition in the region indicated $f(x,\ y) = \frac{\sin y}{x}$, $f(0,\ y) = 0$, $|x| \le 1$, $|y| < \infty$.

3. (a) Compute the first three successive approximations for the solution of the equation $y' = y^2;\quad y(0) = 1$.

   (b) Find an interval I containing Γ and a solution g of $y' = \frac{dy}{dx} = f(x,\ y)$ on I satisfying $g(\Gamma) = s$.

4. (a) Find e^A if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.

   (b) Determine the constants M *and* C and x for the initial value problem $y' = y,\ y(0) = 1$, $R = \{(x,\ y) : |x| \le 1 \text{ and } |y - 1| \le 1\}$.

5. Prove that a necessary and sufficient condition that a solution matrix G be a fundamental matrix is that $G(x) \ne 0$ for $x \in I$.

6. Solve by matrix method the system of equations $\frac{dx_1}{dt} = 9x_1 - 8x_2$; $\frac{dx_2}{dt} = 24x_1 - 8x_2$, where $x_1(0) = 1$ and $x_2(0) = 0$.

7. (a) Find the nature of the critical point (0, 0) of the system $\frac{dx}{dt} = x + 5y$, $\frac{dy}{dt} = 3x + y$ and discuss their stability.

   (b) Explain different type of critical points for a system and give the geometrical meaning of each critical point.

8. (a) Explain the nature of critical point of a non-linear system $\frac{dx}{dt} = ax + by + \phi(x,\ y)$ and $\frac{dy}{dt} = cx + dy + \psi(x,\ y)$.

   (b) Determine the type and stability of the critical point (0, 0) of the non-linear system $\frac{dx}{dt} = Sinx - 4y$; $\frac{dy}{dt} = Sin2x - 5y$.

9. Find the Rodrigue's formula for Legendre polynomial.

10. (a) Derive an expression for the generating function for Bessel's function.

    (b) Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a +ve integer.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VIII
(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that $2^{N_0} = C$, where N_0 is the cardinal number of the set N and C is the cardinal number of [0, 1].
   (b) State and prove Schroder-Bernstein theorem.

2. (a) State Axiom of choice and Zermelo's postulates. Show that Axiom of choice is equivalent to Zermelo's postulates.
   (b) For any three cardinal number α, β, γ; show that (i) $\alpha^{\beta}\alpha^{\gamma} = \alpha^{\beta + \gamma}$, (ii) $(\alpha\,\beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$.

3. (a) Define a countable set. Prove that [0, 1] is uncountable.
   (b) If A and B are two countable sets then show that A × B is also countable.

4. (a) Define isomorphism between two graphs and give two examples of isomorphic graphs.
   (b) Determine the difference between a circuit and Eulerian circuit.

5. (a) What do you mean by a complete graph. Show that a complete graph of n vertices is a planner if $n \leq 4$.
   (b) Prove that a pseudograph is Eulerian iff it is connected and every vertex is even.

6. (a) If g is a connected graph with e-edges and v-vertices, then prove that $e \leq 3v - 6$.
   (b) Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph.

7. (a) State and prove Chinese remainder theorem.
   (b) State and prove the division algorithm of integers.

8. (a) Show that $(a, m_1) = 1, (a, m_2) = 1 \Leftrightarrow (a, m_1 m_2) = 1$.
   (b) Define congruency between two integers under a positive integer m. Prove that the relation $a \equiv b$ (mod m) defines an equivalence relation on the set of integers.

9. (a) State and prove Fermat's theorem.
   (b) If $x \equiv a \pmod 7 \equiv b \pmod{11} \equiv c \pmod{13}$, then prove or disprove that
   $x \equiv -286a + 364b - 77c \pmod{1001}$.

10. (a) What is a circular helix ? Find the osculating plane at the point $P(\theta)$ on the helix $x = a\,Cos\theta,\ y = a\,Sin\,\theta,\ z = c\theta$.
    (b) Prove that $\left[\vec{r}^{\,I}, \vec{r}^{\,II}, \vec{r}^{\,III}\right] = \dfrac{T}{\rho^2}$. Where $\vec{r}$ is the current point, T is torsion and ρ is the radius of curvature.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
Calculator is Allowed.

1. Determine the value of the integral $\int_{4}^{5.2} \log x\, dx$ by Trapezoidal method.

2. Find the formula for Quadrature for equally spaced arguments and hence derive Simpson's three-eighth rule.

3. Solve the equation,
$y_{x+3} - y_{x+2} - y_{x+1} - y_x = 0$, where $y_0 = 2,\ y_1 = -1,\ y_2 = 3$.

4. Form the difference equation corresponding to the family of curves $y_x = ax^2 + bx - 3$.

5. Define factorial notation and prove that $(x)^{(-n)} = \dfrac{1}{(x + hn)^{(n)}}$. Where h is the interval of differencing.

6. (a) Prove that $(1 + \Delta)(1 - \nabla) = 1$.
   (b) Compare Newton's method with Regula-Falsi method. Apply Newton's Raphson method to find square root of 12 to five places of decimals.

7. Compute $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at $x = 2.03$ by Newton's Backward difference formula using the following table :—

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

8. Form Gauss's central difference table and apply it to determine e^{117} from the table :—

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

9. (a) Find f(6) when f(0) = 3, f(1) = 6, f(2) = 8, f(3) = 12 and the third difference being constant.
   (b) Find the positive root of $xe^x - 1 = 0$ lying between 0 and 1 using iteration method.

10. (a) Find all the real roots of the equation $x^2 + 4\,Sin\,x = 0$ correct to four places of decimals.
    (b) Obtain the missing term in the following table :—

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
f(x)	0.135	?	0.111	0.100	?	0.082	0.024

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Examination Programme, 2019

M.Sc. Mathematics, Part-II

Date	***Paper***	***Time***	***Examination Centre***
02.09.2019	Paper–IX	8.00 AM to 11.00 AM	Nalanda Open University, Patna
04.09.2019	Paper–X	8.00 AM to 11.00 AM	Nalanda Open University, Patna
06.09.2019	Paper–XI	8.00 AM to 11.00 AM	Nalanda Open University, Patna
07.09.2019	Paper–XII	8.00 AM to 11.00 AM	Nalanda Open University, Patna
09.09.2019	Paper–XIII	8.00 AM to 11.00 AM	Nalanda Open University, Patna
11.09.2019	Paper–XIV	8.00 AM to 11.00 AM	Nalanda Open University, Patna
13.09.2019	Paper–XV	8.00 AM to 11.00 AM	Nalanda Open University, Patna
16.09.2019	Paper–XVI	8.00 AM to 11.00 AM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–X
(Functional Analysis)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
Calculator is Allowed.

1. Define a normed linear space and a Banach space. In a normed linear space prove that $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.

2. (a) Let X and Y be two normed linear spaces where X is finite dimensional. Then show that every linear map from X to Y is continuous.
   (b) Prove that $x_n \to x \; w.r.t. \; \| \;\; \|$ if and only if $x_n \to x' \; w.r.t. \; \|x\|'$.

3. (a) Let L be a linear space over F, then show that the sum of two inner products on L is also an inner product on L.
   (b) If M and N are closed linear sub spaces of a Hilbert space H such that $M \perp N$ then prove that the linear sub space M + N is also closed.

4. Sate and prove F. Riesz's lemma.

5. State and prove Hahn-Banach theorem.

6. If $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, then prove that dual of l_p is l_q.

7. State polarization identity and explain about it in an inner product space.

8. Give an example of a Banach space which is not a Hilbert space.

9. (a) If the mapping $T \to T'$ is norm preserving mapping of $\beta(N)$ to $\beta(N')$ then prove that,
      (i) $(\alpha T_1 + \beta T_2)' = \alpha T_1' + \beta T_2'$, and
      (ii) $(T_1 T_2)' = T_2' T_1'$.
   (b) If T is a continuous linear transformation of a Banach space X into Banach space Y, then show that T is an open mapping.

10. (a) If H is a Hilbert space, then show that the conjugate space H* is also a Hilbert space.
    (b) If a Hilbert space H is separable, then show that every orthonormal set of H is countable.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XI
(Partial Differential Equations)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Explain Charpit's method for the solution of non-linear partial differential equation of the first order.

2. Using Charpit's method solve the following partial differential equations :—
   (a) $2zx - px^2 - 2qxy + pq = 0$
   (b) $(p^2 + q^2)y = qz$

3. Solve,
   (a) $(D - D'^2)z = Cos\,(x - 3y)$
   (b) $(D^2 - DD' + D' - 1)z = Cos\,(x + 2y) + e^y$.

4. Find the general solution of the partial differential equation $px(x + y) - qy(x + y) = (x - y)(2x + 2y + z)$.

5. By using the method of separation of variables solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

6. Reduce $yr + (x + y)s + xt = 0$ to canonical form and hence solve it.

7. (a) Describe Jacobi's method to solve the partial differential equation $F(x,\ y,\ z,\ p,\ q) = 0$.
   (b) Solve the partial differential equation $xyr + x^2 s - yp = x^3 e^y$.

8. (a) Solve the boundary value problem $\frac{\partial u}{\partial x} = u\frac{\partial u}{\partial y}$, when $u(0,\ y) = 8e^{-3y}$.
   (b) Show that the family of surfaces defined by $x^2 + y^2 =$ constant, is a family of equi-potential surfaces in free space and hence find the law of potential.

9. Solve the boundary value problem $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le t,\ t > 0$ subject to the boundary conditions $\left.\begin{array}{l} u(0,\ t) = 0,\ t > 0 \\ \frac{\partial u}{\partial x}(\ell,\ t) = 0,\ t > 0 \end{array}\right\}$ and the initial conditions $u(x,\ 0) = \begin{cases} x, & 0 \le x < \frac{\ell}{4} \\ \frac{\ell}{2} - x; & \frac{\ell}{4} \le x < \frac{\ell}{2} \\ 0; & \frac{\ell}{2} \le x < \ell \end{cases}$ and $\frac{\partial u}{\partial t}(x,\ 0) = 0,\ 0 \le x < \ell$.

10. (a) Derive the Fourier equation of heat conduction.
    (b) A rod of length ℓ with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to *0°c* and are kept at that temperature. Find the temperature *u (x, t).*

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XII

(Analytical Dynamics)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that Lagrange's Bracket does not obey the commutative law of algebra.

   (b) Prove that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical. Find the generating function *F(q, Q)*.

2. (a) Give the physical significance of Hamilton characteristic function.

   (b) Derive Hamilton-Jacobi equation and then find Hamilton's characteristic function.

3. Discuss the motion of a sphere when the small sphere rolls without slipping on the rough interior of a fixed vertical cylinder of greater radius.

4. Find the equation of motion of simple pendulum applying Lagrange's equation of motion.

5. State and prove Jacobi-Poisson theorem.

6. (a) Describe the motion of particle about revolving axes.

   (b) Using invariance of Bilinear form show that the transformation $Q = \frac{1}{p}$ and $P = p^2 q$ is canonical.

7. Derive the formula for kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of kinetic energy.

8. Derive Lagrange's equation of motion from Hamilton's canonical form of equations.

9. Discuss the motion of spherical pendulum deducing from Hamilton's canonical equations of motion.

10. What do you mean by Hamilton's function ? Find the differential equations for Hamilton's function.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XIII

(Fluid Mechanics)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Derive Euler's equation of motion in cylindrical polar co-ordinates.

2. Derive the equation of continuity in Cartesian form.

3. Describe the motion of a fluid between rotating co-axial circular cylinders.

4. Derive the equation of motion under impulsive force.

5. Prove that the fluid motion is possible when velocity at (x, y, z) is given by $u = \frac{3x^2 - r^2}{r^5}$, $v = \frac{3xy}{r^5}$, $w = \frac{3xz}{r^5}$.

6. A velocity field is given by $\vec{q} = \frac{x\vec{j} - y\vec{i}}{x^2 + y^2}$; calculate the circulation round the square having corners at (1, 0), (2, 0), (2, 1) and (1, 1). Also test for the flow of rotation.

7. (a) Derive the rate of strain tensor of fluid in motion.

   (b) Show that the velocity field defined at a point P by $(1+2y-3z, 4-2x+5z, 6+3x-5y)$ represents a rigid body rotation.

8. (a) What do you mean by Source, Sink and Doublet. Describe them with suitable examples of each.

   (b) A velocity field is given by $\vec{q} = -x\hat{i} + (y+t)\hat{j}$. Find the stream function and the stream lines for field at $t = 2$.

9. (a) Write notes on the following :–

   (i) Velocity Potential

   (ii) Velocity Vector

   (iii) Boundary Surface

   (b) The velocity $\vec{q}$ in a three dimensional flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}$. Determine the equations of streams passing through the point (1, 1, 1).

10. Derive Navier-Stokes equation of motion of viscous fluid.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIV
(Operation Research)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Show that every extreme point of the convex set of feasible solution is a B.F.S. (Basic Feasible Solution).
   (b) Define a convex set in R^n. Let S and T be two convex sets in R^n then show that for any scalars K_1 and K_2, $K_1S + K_2T$ is also a convex set in R^n.
2. (a) Solve the following L.P.P. by any method of your choice (except graphically)
   Max $z = 5x_1 + 7x_2$ s.t.
   $x_1 + x_2 \le 4,\ \ 3x_1 + 8x_2 \le 24,\ \ 10x_1 + 7x_2 \le 35$ *and* $x_1,\ x_2 \ge 0$.
   (b) If (1, 2, 3) is a feasible solution of the set of equations $4x_1 + 2x_2 - 3x_3 = 1$; $6x_1 + 4x_2 - 5x_3 = 1$ then reduce the F.S. to B.F.S. of the set.
3. Solve the following L.P.P. by using two phase simplex method
   Min $z = x_1 + x_2$
   Subject to $2x_1 + x_2 \ge 4,\ \ x_1 + 7x_2 \ge 7;\ \ x_1, x_2 \ge 0$.
4. Find the dual of the following L.P.P.
   Min $z = x_1 + x_2 + x_3$
   Such that $x_1 - 3x_2 + 4x_3 = 5,\ \ x_1 - 2x_2 \le 3,\ \ 2x_2 - x_3 \ge 4;\ \ x_1, x_2 \ge 0$ *and* x_3 is unrestricted in sign.
5. (a) If X_0 and W_0 are feasible solutions to the primal and dual respectively then prove that $c\,X_0 \le W_0 b$.
   (b) Prove that dual of the dual of a given primal is the primal itself.
6. Solve the following L.P.P. problem by simplex method.
   Minimize $z = x_1 - 3x_2 + 2x_3$
   Subject to $3x_1 - x_2 + 2x_3 \le 7,\ -2x_1 + 4x_2 \le 12,\ -4x_1 + 3x_2 + 8x_3 \le 10;\ \ x_1, x_2, x_3 \ge 0$.
7. The pay-off matrix of a game is given below. Find the solution of the game for A and B.

		B				
		I	II	III	IV	V
A	I	-2	0	0	5	3
	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

8. Solve the following L.P.P.
   Max $z = 10x_1 + 3x_2 + 6x_3 + 5x_4$
   S.t. $x_1 + 2x_2 + x_4 \le 6,\ \ 3x_1 + 2x_3 \le 5,\ \ x_2 + 4x_3 + 5x_4 \le 3$ *and* $x_1, x_2, x_3, x_4 \ge 0$
   Also, compute the limits for a_{11} and a_{23} so that the new solution remains optimal feasible solution.
9. Solve the following NLPP using the method of Lagrangian multipliers
   Min $z = x_1^2 + x_2^2 + x_3^2$
   Subject to constraints $4x_1 + x_2^2 + 2x_3 = 14;\ \ x_1, x_2, x_3 \ge 0$.
10. Solve the following assignment problem represented by the following matrix.

	I	II	III	IV	V	VI
A	9	22	58	11	19	97
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	40	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XV
(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove quotient theorem of tensors; give an example.
   (b) What do you mean by symmetric and skew symmetric tensors. Prove that a symmetric tensor of rank two has at most $\frac{1}{2} N(N+1)$ different components in V_N. Where as a skew symmetric tensor of rank two has $\frac{1}{2} N(N-1)$ independent components in V_N.

2. (a) In the matrix notation express the following transformation equations for (i) a covariant vector, (ii) a contravariant vector, (iii) a contravariant tensor of rank two assuming N = 3.
   (b) If a covariant tensor has components xy, $2y - z^2$, zx in rectangular co-ordinates then determine its covariant components in spherical co-ordinates.

3. (a) Prove that the outer product of two tensors (r, s) and (p, q) types is a tensor of $(r + s)(p + q)$ type.
   (b) Show that the co-variant derivative of a co-variant vector is a mixed tensor of rank two.

4. (a) State and prove Convolution theorem for inverse Laplace transform.
   (b) Find the inverse Laplace transform of the following

   (i) $\dfrac{5s + 3}{(s - 1)(s^2 + 2s + 5)}$ (ii) $\dfrac{1}{s^2(s^2 + a^2)}$

5. (a) Prove that the Laplace transform of $\dfrac{Sin\, at}{t}$ is $Cot^{-1}\left(\dfrac{s}{a}\right)$.
   (b) If $L\{F(t)\} = f(s)$ then prove that

   $$L\{F^n(t)\} = s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) \ldots\ldots\ldots\ldots - s F^{n-2}(0) - F^{n-1}(0).$$

6. Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \geq a \end{cases}$ and hence evaluate

   (i) $\displaystyle\int_{-\infty}^{\infty} \frac{Sin\, sa\ Cos\, sx}{s} ds$ (ii) $\displaystyle\int_{0}^{\infty} \frac{Sin\, s}{s} ds$

7. Solve $(D^3 - D^2 + 4D - 4)x = 68 e^t\, Sin\, 2t$. Using Laplace tranform.

8. Form an integral equation corresponding to the differential equation $\dfrac{d^2y}{dx^2} - \sin x \dfrac{dy}{dx} + e^x y = x$ with the initial conditions $y(0) = 1$ and $y'(0) = -1$.

9. Determine deterministic model with instantaneous production. Shortage allowed.

10. (a) Prove that the function $u(t) + (1 + x^2)^{-1/2}$ is a solution of the voltera integral equation.

    $$u(x) = \frac{1}{1 + x^2} - \int_0^x \frac{t}{1 + x^2}\, u(t) dt.$$

    (b) Define Fredholm integral and Voltera integral equations.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XVI
(Programming in 'C')

Annual Examination, 2019

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. What are different types of statements written in C? Explain each type with the help of an example.
2. Explain different types of Scalar data types in C with examples.
3. What is meant by looping in C? Explain some of the looping statements with examples.
4. What is a function? Explain function using an example. Discuss the advantages and disadvantages of using functions.
5. What are different types of If and else statements used in C? Explain each of them with help of an example.
6. Why Switch statements are used in C? How do they differ from other conditional statements? Give an example of Switch statement.
7. What are structures? When and why are they used in C? Give an example to explain them.
8. Differentiate between arrays and pointers. Give examples for each of them.
9. Write short notes on any two of the following :—
   (i) Operators
   (ii) GOTO statement
   (iii) Break and continue statement
   (iv) Global variables.
10. Write a program in C to print the multiplication table of a given number.

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M.Sc. Mathematics, Part–II, Paper–XVI (Practical)
Counselling & Examination Programme, 2019

Practical Counselling Programme

Enrollment No.	***Date***	***Time***	***Venue***
170290001 to 170290525	17.09.2019 to 21.09.2019	10.00 AM to 2.00 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
170290526 to 170290990 & All Old Students		2.00 PM to 6.00 PM	

Practical Examination Programme

Enrollment No.	***Date***	***Time***	***Venue***
170290001 to 170290525	23.09.2019	11.00 AM to 2.00 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
170290526 to 170290990 & All Old Students	23.09.2019	2.30 PM to 5.30 PM	