Nalanda Open University
Annual Examination - 2017
B.Sc. Mathematics (Honours), Part-I
Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. If A, B, C, D are any sets then prove that
   (a) $(A \cup B)x(C \cup D) = (AxC) \cup (BxD) \cup (AxD) \cup (BxC)$
   (b) $(A \cap B)x(C \cap D) = (AxC) \cap (BxD)$
2. State and prove fundamental theorem on equivalence relation.
3. What do you mean by partial order relation and total order relation and well ordered set? Give one example of each.
4. Let $f : A \to B$ and $g : B \to C$ and both f and g are one to one and onto mappings. Then prove that
   (i) $gf : A \to C$ is one to one and onto
   (ii) $(gf)^{-1} = f^{-1} g^{-1}$
5. (a) Define a Lattice, complete Lattice and set an example of Lattice which is not complete Lattice
   (b) Prove that an infinite union of denumerable sets is denumerable.

Group - B

6. (a) Define a group and show that the four fourth roots of unity namely 1, -1, i, -i form a group with respect to multiplication.
   (b) Prove that G = {0, 1, 2, 3, 4, 5} is a finite abelian group of order 6 with respect to addition modulo 6.
7. (a) If a group G has four elements then show that is must be abelian.
   (b) Prove that a group G is abelian if $b^{-1} a^{-1} ba = e \ \forall \ a, b \in G$
8. (a) If H_1 and H_2 are subgroups of a group G then show that $H_1 \cap H_2$ is also a subgroup of G.
   (b) Prove that the order of every element of a finite group is a divisor of the order of the group.

Group - C

9. (a) If A and B are two non-singular matrices of the same order then prove that $(AB)^{-1} = B^{-1} A^{-1}$
   (b) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$
10. Solve the following system of linear equations by matrix method.
    $x + y + z = 6 \qquad 2x + y - 3z = -5 \qquad 3x - 2y + z = 2$
11. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Group - D

12. Find the condition that the equation $x^4 - px^3 - qx^2 + rx + s = 0$ may have its roots in arithmetical progression and solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0.$
13. Solve $x^4 - 3x^2 - 6x - 2 = 0$ by Euler's method.
14. State and prove De-Moivre's theorem.

Examination Programme, 2017 (Revised)
B.Sc (Part – I) All Honours Subjects
Except Home Science, Geography & Statistics Honours

Date	Papers.	Time	Examination Centre
20/3/2017	(Hons) P-I	3.30 to 6.30 pm	Nalanda Open University, Patna
22/3/2017	(Hons) P-II	3.30 to 6.30 pm	Nalanda Open University, Patna
24/3/2017	Rastrabhsha-100 or Hindi+Urdu 100	3.30 to 6.30 pm	Nalanda Open University, Patna
27/3/2017	Math (Sub) P-I	8.00 to 11.00 am	Nalanda Open University, Patna
28/3/2017	Geography (Sub) P-I	8.00 to 11.00 am	Nalanda Open University, Patna
29/3/2017	Chemistry (Sub) P-I	8.00 to 11.00 am	Nalanda Open University, Patna
30/3/2017	Home Science (Sub)-P I	8.00 to 11.00 am	Nalanda Open University, Patna
31/3/2017	Zoology (Sub) P-I	8.00 to 11.00 am	Nalanda Open University, Patna
03/4/2017	Physics (Sub) P-I	8.00 to 11.00 am	Nalanda Open University, Patna
04/4/2017	Botany (Sub) P-I	8.00 to 11.00 am	Nalanda Open University, Patna
05/4/2017	Statistics (Sub) P-I	8.00 to 11.00 am	Nalanda Open University, Patna

Nalanda Open University
Annual Examination - 2017
B.Sc. Mathematics (Honours), Part-I
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) If $y = \sin(ax+b)$ then find yn

(b) If $y = e^{a\sin^{-1}x}$, then prove that
$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2+a^2)yn = 0$

2. (a) State and prove Taylor's Theorem.

(b) Prove that $e^{a\sin x} = 1 + x + \frac{1}{2}x^2 - \frac{1}{4}x^3 - \frac{1}{2}.\frac{1}{4}x^4 + ...\infty$

3. (a) Evaluate $\underset{x\to 0}{Lt.} \frac{xe^x - \log(1+x)}{x^2}$

(b) $\underset{x\to 0}{Lt.} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

4. (a) If $u = \log(x^2+y^2+z^2-3xyz)$ then show the $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x^2+y^2+z^2)^2}$

(b) If the normal at any point to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ make an angle ϕ with the x-axis then show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$

5. (a) find the radius of curvature for the pedal curve *p=f(r)* or prove that $\rho = r\frac{dr}{dp}$, symbols have usual meaning.

(b) Find the asymptotes to the cure $(x^2+y^2)(x+2y+2) = x+9y+2$

Group - B

6. Evaluate any two of the following:

(a) $\int \frac{x^2 dx}{x^4+1}$ (b) $\int \frac{dx}{x^3+a^3}$ (c) $\int \frac{x^5 dx}{x^9-1}$

7. (a) Show that $\int_0^\pi \frac{xdx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi^2}{2ab}$

(b) Show that $\int_0^{\pi/2} = \log(\sin x)dx = \frac{\pi^2}{2}\log\left(\frac{1}{2}\right)$

8. (a) Evaluate $\underset{n\to\infty}{Lt.} \left[\frac{1^2}{1^3+n^3} + \frac{2^2}{2^3+n^3} + ...\frac{n^2}{n^3+n^3}\right]$

(b) Obtain the reduction formula for $\int \cos^m x \sin nx dx$

9. (a) Find the area of the loop of the curve $xy^2 + (x+a)^2(x+2a) = 0$

10. Find the surface of the solid obtained by revolving the curve $r^2 = a^2\cos 2\theta$ about the initial line.

Group – C

11. (a) Find the polar equation of the conic in the form $\frac{l}{r} = 1 + e\cos\theta$

(b) Find the polar equation of tangent at any point of it to the covic $\frac{l}{r} = 1 + e\cos\theta$

12. (a) Deduce the equation of a plane in the form $lx + my + nz = p$

(b) Prove that $\frac{a}{y-z} + \frac{b}{z-x} + \frac{c}{x-y} = 0$ represents a pair of plains.

13. (a) Find the equation to the perpendicular from the origin to the line $x+4y+4z-27=0$; $2x+2y+5z-2=0$ also find the co-ordinates of the foot of perpendicular.

(b) Find the image of the point $(1,2,3)$ by the plane $x+y+z=3$

14. (a) Find the equation of the sphere through the circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ and the point $(1,2,3)$

(b) Find the pole of the plane $lx+my+nz=p$ with respect to the sphere $x^2+y^2+z^2=a^2$

Nalanda Open University

Annual Examination - 2017

B.Sc. Mathematics (Subsidiary), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting atleast **One** question from each group.*
All questions carry equals marks.

Group - A

1. If {Ai} i ∈ I be an arbitrary indexed family of sets then show that:

   (a) $\left(\underset{i\in I}{\quad} A_i\right)' = \underset{i\in I}{\quad} A_i'$ (b) $\left(\underset{i\in I}{\quad} A_i\right)' = \underset{i\in I}{\quad} A_i'$

2. (a) Prove that $A x (B \quad C) = (AxB) \quad (AxC)$

   (b) Prove that $A x (B \quad C) = (AxB) \quad (AxC)$

3. (a) Define an equivalance relation and give two examples of it.

   (b) If $f : X \to Y, A \subseteq Y, B \subseteq Y$ then show the $f^{-1}(A \quad B) = f'(A) \quad f^{-1}(B)$

4. (a) Show that the set G={1, w, w^2} where w is an imaginary cube root of unity is a group with respect to multiplication.

   (b) Prove that the order of every element of a finite group is finite and is less than or equal to the order of the group.

5. (a) Define an abelian group and show that if a group G has four elements it must be abelian.

   (b) Prove that a necessary and sufficient condition for a non-empty subset H of group G to be a sub group of G is that $a \in H, b \in H \Rightarrow ab^{-1} \in H$. Where b^{-1} is the inverge of b in G

6. (a) If *f* is a homomorphism of group G into a group G′ then prove that

   (i) *f(e)=e′* where e is the identity of G and e′ is the identity of G′

   (ii) $(a^{-1}) = [f(a)]^{-1}$ ∀a ∈ G

   (iii) If the order of a ∈ G is finite then order of f(a) is the divisor of the order of a.

   (b) If *f* is a homomorphism of a group G into a group G′ with kernel K = {*x* ∈ *G* : *f(x)* = *e′* is the identity of G′ } then show that K is a normal subgroup of G.

Group - B

7. (a) Extract all the roots of $x^7+1 = 0$ using De-Moivre's Theorem.

   (b) Decompose $\log(\alpha + i\beta)$ into real and imaginary parts.

8. (a) State and prove Cauchy general principle of convergence of a real sequence.

   (b) Show that the sequence (a_n) where $a_n = \sqrt{n^2+4n} - n$ is convergent.

9. (a) Prove that the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + ...\infty$ is convergent for p > 1 and divergnet for p ≤ 1.

   (b) Test the convergence of the series whose n^{th} term is $\sqrt{n^2+1} - \sqrt{n^2-1}$

Group - C

10. Find the condition under which a general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse.

11. Deduce the polar equation of a conic in the form $\frac{l}{r} = 1 + e\cos\theta$.

12. (a) State and prove Euler's theorem on Homogeneous functions of two variables.

    (b) If $f(x, y) = x\cos y + y\cos x$ then prove that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

13. (a) Prove that the radius of curvature for the polar curve $r = f(\theta)$ is given by

$$\rho = \left(\frac{r^2 + {r_1}^2)^{3/2}}{r^2 + 2{r_1}^2 - rr_2} \right)$$

(b) Find the maximum value of $x^{\frac{1}{x}}$

14. (a) Give the geometrical meaning of scalar triple product of vectors.

(b) Prove that $ax(bxc) = (a.c)b - (a.b)c$.

Nalanda Open University
Annual Examination - 2017
B.Sc. Mathematics (Honours), Part-II
Paper-III

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) State and prove theorem of greatest lower bound.
   (b) State and prove theorem of least upper bound.
2. (a) State and prove Dedekind's theorem for real numbers.
   (b) State and prove fundamental theorem of classical analysis.
3. Prove that between two distinct real numbers there lie infinity of rationals and infinity of irrationals

Group-B

4. (a) Prove that every convergent squence is bounded.
   (b) Prove that a monotonic increasing sequence tends to its bound.
5. (a) Prove that the sequence defined by $u_1= \sqrt{2}$, $u_{n+1}=\sqrt{2u_n}$ converges to 2.
   (b) Show that the sequence (a_n) defined by $a_1=\sqrt{7}$, $a_n+1=\sqrt{7+a_n}$ converges to a positive root of the equation $x^2 - x - 7 = 0$.
6. (a) Test the convergence of the series
   $$\frac{1}{1^p}+\frac{1}{3^p}+\frac{1}{5^p}+\frac{1}{7^p}+.............\infty$$
   (b) Test the convergence of the series whose n^{th} term is $\sqrt{n^2+1}-\sqrt{n^2-1}$
7. (a) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{x^n}{1+n^2}$; for all $x > 0$
   (b) State and prove Couchy's n^{th} root test for convergence of an infinite series.
8. (a) State and prove Raabe's test for convergence of the positive term series Σu_n.
   (b) Test the convergence of the series $\sum\frac{(n+1)(n+2)}{(n+3)(n+4)}$

Group-C

9. (a) Prove that is W_1 and W_2 are finite dimensional subspaces of a vector space V, then W_1+W_2 is a finite dimensional and $\dim.W_1 + \dim.W_2 = \dim (W_1 \cap W_2) + \dim(W_1 + W_2)$
10. (a) Find the eigen values and eigen vectors of the matix $A=\begin{bmatrix}-4 & -6\\ 3 & 5\end{bmatrix}$
    (b) Find the rank of the matrix $\begin{bmatrix}1 & 2 & -1\\ 3 & -1 & 2\\ 2 & -2 & 3\\ 1 & -1 & 1\end{bmatrix}$

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Examination Programme, 2017
(Bachelor of Science (Part-II)
All Subjects Except B.Sc Geography & Home Science (Honours)

Date	Paper	Time	Name of Examination Centre
23/2/2017	HONOURS PAPER – III	8.00 to 11.00 am	Nalanda Open University, Patna
25/2/2017	HONOURS PAPER – IV	8.00 to 11.00 am	Nalanda Open University, Patna
27/2/2017	Hindi 100 or Ur 50+Hn 50	8.00 to 11.00 am	Nalanda Open University, Patna
01/3/2017	(SUB.) (Botany - II)	8.00 to 11.00 am	Nalanda Open University, Patna
02/3/2017	(SUB.) (Mathematics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
04/3/2017	(SUB.) (Chemistry - II)	8.00 to 11.00 am	Nalanda Open University, Patna
06/3/2017	(SUB.) (Physics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
07/3/2017	(SUB.) (Zoology - II)	8.00 to 11.00 am	Nalanda Open University, Patna
09/3/2017	(SUB.) (Geography - II)	8.00 to 11.00 am	Nalanda Open University, Patna
11/3/2017	(SUB.) (Home Science- II)	8.00 to 11.00 am	Nalanda Open University, Patna
16/3/2017	(SUB) Statistics-II)	8.00 to 11.00 am	Nalanda Open University, Patna

Nalanda Open University
Annual Examination - 2017
B.Sc. Mathematics (Honours), Part-II
Paper-IV

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one qu estion from each group.

Group-A

1. (a) Solve any two of the following differential equations
(a) $(px-y)(x-py)=2p$ (b) $(x-a)p^2 + (x-y)p - y=0$

2. (a) Prove that the system of confocal conic $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal.
(b) Examine the equation $x^3p^2 + x^2py + a^3 = 0$ for singular solutions.

3. (a) Solve $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+4x^2 y = x^4$ (use method of change of variable)
(b) Using method of variation of parameter solve $\frac{d^2y}{dx^2}+a^2y=\sec ax$ OR solve $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$ (using method of variation of parameter)

Group-B

4. (a) Prove that $\vec{a}\,x\,(\vec{b}\,x\,\vec{c}\,)=(\vec{a}\,.\,\vec{c})\vec{b}-(\vec{a}\,.\,\vec{b})\,\vec{c}$
(b) Show that $[\vec{a}\,x\,\vec{b}\quad \vec{b}\,x\,\vec{c}\quad \vec{c}\,x\,\vec{a}]=[\vec{a}\,\vec{b}\,\vec{c}]^2$

5. (a) Prove that $\frac{d}{dt}(\vec{u}\,x\,\vec{v})=\vec{u}\,x\frac{d\vec{v}}{dt}+\frac{d\vec{u}}{dt}x\,\vec{v}$
(b) Find a unit tangent vector to any point on the space curve $x = a\cos t,\ y = a\sin t,\ z = bt$ where a, b are constants and t is time.

6. (a) Prove the $\nabla\,x\,(\vec{u}\pm\vec{v})=\nabla x\,\vec{u}\pm\nabla x\,\vec{v}$
(b) Prove that $\nabla.(\vec{u}\,x\,\vec{v})=\vec{v}.(\nabla x\,\vec{u})-\vec{u}.(\nabla x\,\vec{v})$

Group-C

7. State and prove the necessary and sufficient condition for equilibrium of a system of co-planar forces.
8. (a) State and prove the necessary and sufficient condition of the principle of virtual work.

Group-D

9. Derive the tangential and normal acceleration in polar co-ordinates.
10. If in a simple Harmonic motion u, v, w be the velocities at distance a, b, c from a fixed point on the straight line which is not the centre of force, show that the periodic time T is given by the equation

$$4\pi^2(b-c)(c-a)(a-b)=T\begin{vmatrix}u^2 & v^2 & w^2\\ a & b & c\\ 1 & 1 & 1\end{vmatrix}.$$

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Nalanda Open University
Annual Examination - 2017
B.Sc. Mathematics (Subsidiary), Part-II
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Eight questions, selecting atleast one from each group. All questions carry equal marks.

Group-A

1. Evaluate any two of the following:

(a) $\int e^x \frac{(1+x^2)}{(1+x)^2} dx$ (b) $\int \frac{1}{x^3+1} dx$ (c) $\int \frac{dx}{\sin x(3+2\cos x)}$

2. Evaluate any two

(a) $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x} dx$ (b) $\int_0^{\pi} \frac{xdx}{(a^2\cos^2 x+b^2\sin^2 x)^2}$ (c) $\int_0^{\pi/2} \log(\tan x) dx$

3. (a) Evaluate $\underset{n\to\infty}{Lt.}\left[\frac{1^2}{1^3+n^3}+\frac{2^2}{2^3+n^3}+\text{..........}+\frac{n^2}{n^3+n^3}\right]$

(b) Find the reduction formula for $\int \cos^m x \sin^n x dx$

4. (a) Find the area between the curve $y^2(a+x)=(a-x)^3$ and its asymptote.

5. Find the perimeter of the loop of the curve $9ay^2=(x-2a)(x-5a)^2$

6. Find the volume formed by the revolution of the loop of the curve $y^2(a+x)=x^2(a-x)$ about the x-axis.

7. Solve the following differential equations :

(a) $y=2px+ap^2$ (b) $y+px=x^4p^2$

8. Solve the following differential equations:

(a) $\frac{d^2y}{dx^2}+4y=x^2$ (b) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$

Group – B

9. Find the length and the equation of the shortest distance between two skew lines

$$\frac{x-\alpha}{}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}\ and\ \frac{x-\alpha^1}{1}=\frac{y-\beta^1}{m^1}=\frac{z-\gamma^1}{n^1}$$

10. Find the volume of the Tetrahedron the co-ordinates of whose vertices are given.

11. (a) Define a convex set and a hyper plane and prove that a hyper plane is a convex set.

(b) Prove that the intersection of a finite number of convex sets is a conves set.

Group – C

12. State and prove principle of virtual work.

13. Find the necessary and sufficient condition for the euqilibrium of a system of co-planar forces.

14. (a) What do you mean by S.H.M., derive an expression for time period.

(b) Discuss the motion of a body under inverse square law in details.

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Nalanda Open University

Annual Examination - 2017

B.Sc. Mathematics (Honours), Part-III

Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) State and prove MinKowski inequality.
2. If d is a metric for X, show that the function defined by d^* : X x X → R defined as $d^* = \frac{d(x,y)}{1+d(x,y)}$ is also a metic for X.
3. If $1 < \rho < \infty, 1 < q < \infty$ such that $\frac{1}{p} + \frac{1}{q} = 1$ and a,b are real numbers such that a ≥ 0, b ≥ 0 then prove that $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$ or $a^{y/p} b^{\frac{1}{q}} \leq \frac{a}{p} + \frac{b}{q}$
4. (a) Prove that in a metric space (x, d) each open sphere is an open set.
5. (a) Prove that every metric space is T_2-space.
   (b) Prove that every metric space is first countable.

Group 'B'

6. What do you mean by a Hausdorff space. Show that every discrete topological space is Hausdorff.
7. Let (X,T) be a topological space and A and B are subsets of X. If $\overline{A}$ denotes closure of A then show that
   (a) $(\overline{A \cap B}) = \overline{A} \cap \overline{B}$ (b) $(\overline{A \cup B}) = \overline{A} \cup \overline{B}$ (c) $\overline{\overline{A}} = \overline{A}$

Group 'C'

8. State and prove Darboux's theorem.
9. Prove that if a bounded function f is R-integrable over [a,b] and M and m are bounds of f then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$ $if\ b \geq a.$

Group 'D'

10. Test the covergence of the series $\sum_{m=1}^{\infty} \frac{1}{n \log n (\log \log n)^p}$
11. Show that the sum of the series $1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \frac{1}{5} - \frac{1}{10} - \frac{1}{12} + ...$ is half the sum of the series $1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + ...$

Examination Programme-2017

B.Sc (Part–III) Botany, Chemistry, Mathematics, Physics & Zoology Honours

Date	Papers	Time	Examination Centre
14/2/2017	Honours Paper–V	8 to 11 AM	Nalanda Open University, Patna
16/2/2017	Honours Paper–VI	8 to 11 AM	Nalanda Open University, Patna
18/2/2017	Honours Paper–VII	8 to 11 AM	Nalanda Open University, Patna
20/2/2017	Honours Paper–VIII	8 to 11 AM	Nalanda Open University, Patna
22/2/2017	Paper –XV (General Studies)	8 to 11 AM	Nalanda Open University, Patna

Nalanda Open University
Annual Examination - 2017
B.Sc. Mathematics (Honours), Part-III
Paper-VI

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) *Define a ring homo morphism. Let $f:R \rightarrow R^1$ be a homomorphism of ring R onto a ring R^1 then show that f is a isomorphism iff kernel f={0}*
   (b) *Show by an example that if I_1 and I_2 are ideals of a ring R then I_1, UI_2 is note an ideal of R.*
2. (a) *Show that any ring can be embedded in a ring with unity.*
   (b) *Define a principal ideal ring and show that the ring of integers is a principal ideal ring.*
3. (a) *define an automorphism of a group G. Let $x \in G$ then prove that the function f defined by f (g) $=x^{-1}gx$ for g in G is an automorphism of G.*
   (b) *Let G be a group. Then for any element g in G, porve that $C_o(g)$ is a Sub group of G.*

Group 'B'

4. (a) *Give examples of two polynomials f(x) and g(x) such that deg (fg) < deg (f) + deg (g)*
   (b) *Prove that the set of all polynomials in Z[x] with constant term O is prime ideal in Z[x].*
5. (a) *What is a prime field? prove that Q the set of all national numbers is a prime field.*
   (b) *Let R[x] be the set of all polynomials where $a_0, a_1, a_2, \ldots\ldots\ldots$, am $\in$ R and $_a m \neq 0$ as well, m is a non-negaive integer. If R is a commutative ring with unity then show that R [x] is also a commutative ring with unity.*

Group 'C'

6. *State and prove Cantor's theorem.*
7. (a) *Prove that $2^{\lambda^\circ} = C$ (Symbols have their usual meaning)*
   (b) *For any three Cardinal numbers $\propto, \beta, \gamma$ prove that*
   (i) $\propto^{\beta} \propto^{\gamma} = \propto^{\beta+\gamma}$ (ii) $(\propto \cdot \beta)^{\gamma} = \propto^{\gamma} \beta^{\gamma}$ (iii) $(\propto^{\beta})^{\gamma} = \propto^{\beta\gamma}$
8. (a) *Let X be any non-empty set. Then show that Card (P(X) is 2 were PX is the power set of X.*
   (b) *Introducee the Concept of order types and Construct the product of two order types.*

Group 'D'

9. *What are the neassary and sufficient Condition for differentiability of a Complext valued function.*
10. *Show that the function f(z) = $\sqrt{|xy|}$ is not analytic at the origin, although Cauchy Riemann differential equations are satisfied.*
11. (a) *State and prove Cauchy integral formula.*
    (b) *Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle I Z I = 3*

Nalanda Open University

Annual Examination - 2017

B.Sc. Mathematics (Honours), Part-III

(Graphpaper may be supplied)

Paper-VII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. Use simplex method to solve the following L.P.P.
   Min: $x_2 - 3x_3 + 2x_5$
   Subject to $x_1 + 3x_2 - x_3 + 2x_5 = 7$, $-2x_2 + 4x_3 + x_4 = 12$, $x_j \geq 0$; $j = 1, 2, 3, 4, 5, 6$
2. Solve the following L.P.P. graphically
   Minimize $z = x_1 + 2x_2$
   Subject to $x_1 - 3x_2 \leq 6$, $2x_1 + 4x_2 \geq 8$, $x_1 - 3x_2 \geq -6$, $x_1, x_2 \geq 0$
3. (a) Define a covex set, the subset of R^n and show that the finite intersection of convex sets is covex set.
   (b) Prove that every hyperplane is convex.

Group 'B'

4. (a) Solve $\frac{dx}{dt} + 4x + 3y = t$, $\frac{dy}{dt} + 2x + 5y = e^t$
   (b) Solve $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$.
5. Solve (a) $(y+z)p + (z+x)q = x+y$ (b) $pz - qz = z^2 + (x+y)^2$
6. Using Charpit's method to solve $(p^2+q^2)x = pz$

Group 'C'

7. Find the attraction of a uniform sphere at an external point of it.
8. State and prove Laplace theorem in Cartesian form.

Group 'D'

9. (a) Find the depth of the centre of pressure of a triangle immersed in a liquid with the vertex in the surface and base horizontal.
   (b) Find the depth of centre of pressure of a circular area of radius 'a' immersed vertically in a homogeneous fluid.
10. (a) A rod of small cross-section and density ρ has a small portion of metal of weight $\frac{1}{n}$th that of the rod attached to one extremity. Prove that the rod will float at an engle in a liquid of density σ if $(n+1)^2 \rho = n^2 \sigma$.
    (b) Prove that the difference of pressures at two points of a homogeneous fluid varies as the depth of one point below the other.

Nalanda Open University
Annual Examination - 2017
B.Sc. Mathematics (Honours), Part-III
Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Five questions. All questions carry equal marks.

1. (a) *Describe Euler's method of solution for differential equation and hence find approximate value of y for x=0, 1. Given that* $\frac{dy}{dx} = \frac{y-x}{y+x}$ *when y=1 for x = 0.*

2. (a) *Use Gauss-Jordan's method to solve the system of linear equations* $x_1+2x_2+x_3=8$, $2x_1+3x_2+4x_3=20$ *and* $4x_1+3x_2+3x_3=16$ *taking initial condition x=0, y=0, z=0*

3. *Apply analytic method for finding roots of an equation based on Rolle's theorem and demonstrate on* $3x-\sqrt{1+\sin x}=0$

4. (a) *Derive Simpson's* $\frac{3}{8}th$ *rule for numerical integration.*

   (b) *By using Weddle's rule evaluate* $\int_0^{10} \frac{dx}{1+x}$.

5. *Use group relaxation method to solve the following system of equations -10x+2y+4z+4=0, x-10y+2z+10=0, x+y-10z+45=0 (Take initial condition x=0, y=0, z=0)*

6. (a) *Discuss Newton-Raphson's method to obtain approximate value of root of f(x)=0*

   (b) *Use synthetic division to solve* $f(x)=x^3-x^2-(1.001)x+0.9999=0$ *in the neighbourhood of x=1.*

7. (a) *Explain the meaning of the operators E and Δ and show that E and Δ are commutative with respect to variables.*

   (b) *Evaluate (i)* $\Delta^3(1-x)(1-2x)(1-3x)$ *and (ii)* $\Delta^n(e^{ax+b})$ *where a and b are constant.*

8. (a) *Describe Newton-Gregory formula for backward interpolation.*

9. (a) *Describe Picard's method of successive approximation.*

   (b) *Apply Runge-Kutta method for the solution of first order differential equation.*

10. (a) *Explain Gauss's method of elimination for the solution of a system of m equations in m variables.*

    (b) *Solve the following system of equations* $x_1+\frac{1}{2}.x_2+\frac{1}{3}.x_3=1,\ \frac{1}{2}.x_1+\frac{1}{3}.x_2+\frac{1}{4}.x_3=0,\ \frac{1}{3}.x_1+\frac{1}{4}.x_2+\frac{1}{5}.x_3=0$