NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–I
(Advanced Abstract Algebra)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a sub-normal series of a group. Hence or ortherwise form a sub-normal series of the additive group of integers.
   (b) Construct all composition series of Z_{60}.
2. State and prove Jordan-Holder theorem on any group.
3. If a, b, c are three non-zero elements of an Euclidean space R such that a and b are relatively prime and a/bc, then prove that a/c.
4. Establish the transitivity property of finite extension of a field.
5. (a) Prove that the range of homomorphism of a module is a sub-module of the module.
   (b) Prove that is every principal ideal domain, each pair of elements has a greatest common divisor.
6. Define Homomorphism and kernel of homomorphism from a module M into a module N. If f is a module homomorphism then f is an isomorphism *iff* $K(f)=0$. Prove this statement.
7. (a) Define algebraic and simple extension of a field and give one example of each of them.
   (b) If a and b are algebraic over a field F, then prove that $a \pm b,\ ab,\ ab^{-1}\left(b \neq 0\right)$ are also algebraic over F.
8. State and prove Kronecker's theorem.
9. (a) Prove that, if $K=\phi\left(\sqrt{2}\right)$, where ϕ is the field of all rational numbers, then ϕ is the fixed field under the group of automorphism of K.
   (b) If K is the field of complex number and F is the field of real numbers, then show that K is normal extension of F.
10. State and prove fundamental theorem of Galois Theory.

* * *

Examination Programme, 2015

M.Sc. Mathematics, Part–I

Date	*Paper*	*Time*	*Examination Centre*
11.05.2015	Paper–I	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna
13.05.2015	Paper–II	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna
15.05.2015	Paper–III	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna
19.05.2015	Paper–IV	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna
21.05.2015	Paper–V	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna
23.05.2015	Paper–VI	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna
25.05.2015	Paper–VII	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna
27.05.2015	Paper–VIII	3.30 PM to 6.30 PM	MGM College, Kankarbagh Main Road, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–II
(Real Analysis)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) State and verify Bolzano-Weierstrass theorem by a suitable example.
   (b) Deduce Bolzano-Weierstrass theorem from Heine-Borel theorem.

2. Show that a function *f* on [a, b] is of bounded variation iff it can be represented as a difference of two monotonically increasing functions on [a, b].

3. (a) State and prove a necessary and sufficient condition for $f \in R(\alpha)$ on [a, b].
   (b) Prove that if $\lim\limits_{\|P\| \to 0} S(P, f, \alpha)$ exists, then *f* is integrable in the sense of Stieltjes over [a, b] with respect to α and $\lim\limits_{\|P\| \to 0} S(P, f, \alpha) = \int_a^b f(x)\alpha'(x)dx$.

4. (a) If $f \in R(\alpha)$ and α has continuous derivative α' on [a, b], then prove that Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists and $\int_a^b f(x)d\,\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.
   (b) If $f \in R(\alpha)$ on [a, b] and if $\int_a^b f d\alpha = 0$, for every *f*, which is monotonic on [a, b], then prove that α must be constant on [a, b].

5. (a) Let *f* be a vector-valued function defined on an open set E of R^n with values in R^m. If both partial derivatives $D_i f$ and $D_j f$ exist on n-ball $B(C, \delta)$ and if both are differentiable at C, then prove that $D_{i,j} f(c) = D_{j,i} f(c)$.
   (b) Let $f : R^2 \to R$ be defined by $f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}$, if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$, then show that $D_{1,2}\ f(0, 0) \neq D_{2,1}\ f(0, 0)$.

6. (a) Let E be an open sub-set of R^n and *f* be a vector valued function $f : E \to R^m$. If *f* is differentiable at $\underline{C}$ of E, then prove that *f* is continuous at $\underline{C}$.
   (b) Let $f = (f_1, f_2, ---, f_m)$ be a vector valued function defined on an open sub-set E of R^n with values in R^m. Define continuity, partial derivatives and directional derivative of *f* in E.

7. (a) State and prove Abel's theorem.
   (b) Find the radius of convergence of the following power series $\sum_{n=1}^{\infty} \frac{\lfloor n}{n^n} x^n$.

8. (a) Explain extreme values of a function and find these values for the function $f(x, y, z) = 2xyz - 4zx - 2yz + x^2 + y^2 + z^2 - 2x - 4y + 4z$.
   (b) Find Taylor's formula for functions from R^n to R^m.

9. State and prove Inverse Function theorem.

10. Find $\frac{\partial(y_1, y_2, ---, y_n)}{\partial(x_1, x_2, ---, x_n)}$, where $y_1 = x_1(1 - x_2)$, $y_2 = x_1 x_2 (1 - x_3)$, $y_3 = x_1 x_2 x_3 (1 - x_4)$, ---, $y_{n-1} = x_1 x_2 --- x_{n-1}(1 - x_n)$, $y_n = x_1 x_2 --- x_n$.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–III
(Measure Theory)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) If E_1 and E_2 are measurable, sets, then show that $E_1 \cup E_2$ is measurable.
   (b) Show that an enumerable set is measurable with measure zero.

2. Let $(E_i)_{i \in N}$ be an infinite decreasing sequence of measurable sets and $m(E_i) < \propto$. Prove that $m\left(\bigcap_{i=1}^{\propto} E_i\right) = \lim_{i \to \propto} m(E_i)$.

3. (a) Prove that a continuous function is measurable but every measurable function is not continuous.
   (b) If a function *f* is measurable, then show that $|f|$ is measurable.

4. If *f* is integrable, then show that $|f|$ is integrable and conversely with $\left|\int f\right| \leq \int |f|$.

5. (a) Define integrability of a measurable function. Show that if *f* and *g* are integrable then *f* + *g* is integrable and $\int (f+g) = \int f + \int g$.
   (b) If *f* is integrable, then show that *f* = *a.e.* finite.

6. Examine the L-integrability of $f(x) = \frac{d}{dx}\left(x^2 \sin \frac{1}{x^2}\right)$ over [0, 1].

7. State and prove Fatou's lemma.

8. (a) State and prove Lebesgue monotone convergence theorem.
   (b) Verify bounded convergence theorem for $f_n(x) = \frac{nx}{1+n^2x^2} (0 \leq x \leq 1), n = 1, 2, 3, ---$

9. Compare integrabilities in the sense of Riemann and Lebesque. Prove that every function, which is *R*-integrable is also *L*-integrable on [a, b], but the converse is not true (use counter example).

10. (a) Show that an absolutely continuous function is continuous, but not the converse.
    (b) If *f* is integrable over [a, b] and $\int_a^x f(t)dt = 0$ for all *x* in [a, b], then prove that $f(x) = 0$ a.e. in [a, b].

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–IV
(Topology)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a topological space X and $A \subseteq X$ be a closed set. Then show that A is the disjoint union of its set of isolated points and its set of limit points.
   (b) Show that a set A is open iff A is disjoint from its boundary.
2. (a) Let x be a topological space and A, B be subsets of x. Then prove that $Int(A) \cap Int(B) = Int(A \cap B)$.
   (b) Show that the boundary of a closed set is no where dense.
3. (a) Show that in a Housdorff space, every convergent sequence has a unique limit.
   (b) Show that the product of any non-empty class of Hausdorff spaces is a Housdorff space (Give an example).
4. Prove that a necessary and sufficient condition for a one-to-one mapping $f : x \rightarrow y$ (x and y being topological spaces) to be a homeomorphism is that $f(\overline{A}) = \overline{f(A)}$ for every $A \subseteq x$.
5. (a) Show that a topological space is T_0-Space, *iff* $x, y \in X$ and $x \neq y \Rightarrow \overline{\{x\}} \neq \overline{\{y\}}$.
   (b) Show that the property of a T_1-space is both hereditary and topological.
6. (a) Prove that a topological space is normal *iff* each neghourhood of a closed set F contains the closure of some heghbourhoods of F.
   (b) Show that every metric space is a normal space.
7. (a) Prove that every compact sub-space of a Hausdorff space is closed.
   (b) Show that the open interval (0, 1) on the real line R is not compact.
8. (a) Show that every compact sub-space of the real line is closed and bounded.
   (b) Prove that a compact Housdorff space is regular.
9. (a) Introduce the concept of connected and disconnected spaces. Prove that a topological space x is connected *iff* ϕ and x are its only sub-sets which are both open and closed.
   (b) Show that the connectedness is not a hereditary property.
10. If X and Y are topological spaces, then show that X × Y is connected *iff* X and Y are connected.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–V
(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that a linear operator E is a projection on some sub-space iff it is an idempotent.
   (b) If E is a projection on M along N, then show that $I\text{-}E$ is a projection on N along M and vice versa, where V is the direct sum of sub-spaces M and N,

2. Let $V(F)$ be a finite dimensional vector space and W is a sub-space of V, then prove that $\dim V/W = \dim V - \dim W$.

3. (a) If f is a linear functional on a vector space $V(K)$, then show that (i) $f(0) = 0$ and $f(-x) = -f(x)$ for all $x \in V$.
   (b) Prove that two real quadratic forms are equivalent iff they have the same rank and index.

4. Define linear transformation and its null space. If $U(f)$ and $V(f)$ are two vector spaces and T is a linear transformation from U to V, then show that the kernel T or null space of T, is a sub-space of U.

5. (a) Show that the partial ordered set $(P(x), \subseteq)$ is a lattice.
   (b) Let R be a ring and L be the lattice of all ideals of R, them prove that L is a modular.

6. (a) Define isomorphism between two lattices.
   (b) The necessary and sufficient conditions for a one-one onto mapping between two lattices to be an isomorphism, is that f and f^{-1} are both order preserving.

7. Prove that clopen sets (open and closed) G having $0 = \phi$ and $1 = x$, is a Boolean Algebra.

8. (a) $\forall\, x, y \in B$ (Boolean Algebra) the following are equivalent (i) $x \wedge y' = 0$, (ii) $x \vee y = y$, (iii) $x' \vee y = 1$, (iv) $x \wedge y = x$. Prove this.
   (b) Show that the relation preceeds $(x \le y)$ in a Boolean Algebra B is a partial order relation in B.

9. (a) Prove that in Boolean Algebra, the complement of an element is unique.
   (b) Prove that a Boolean Algebra B is a complement distributive lattice.

10. (a) Prove that the determinant of a unitary operator has absolute value 1.
    (b) Show that the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent of index 3.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VI
(Complex Analysis)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Derive the sufficient condition for analyticity of a function $f(z)$.

   (b) Show that $u = ½\log(x^2 + y^2)$ is a harmonic function. Find the analytic function $f(z)$ whose real part is u.

2. (a) What is a power series ? State and prove Cauchy-Hadmard theorem.

   (b) Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{2^n + 1}$.

3. (a) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i$ and $z_3 = -2$ to the points $w_1 = 1,\ w_2 = 1$ and $w_3 = -1$ respectively.

   (b) If $w = z + z^2$, prove that the circle $|z| = 1$ correspond to a cardoid in w-plane.

4. Define cross ratio of numbers. Prove that the cross ratio remains invariant under a bilinear transformation.

5. State and prove Cauchy's theorem. Describe its application in one or two cases.

6. State and prove Poisson's Integral formula.

7. Using Cauchy's Integral formula, evaluate any one of the following integrals :—

   (a) $\int_C \frac{z\,dz}{(9 - z^2)(z + i)}$, where C is the circle described anticlockwise and having equation $|z| = 2$.

   (b) $\int_C \frac{Cos\hbar\ (\pi z)dz}{z(z^2 + 1)}$, where C is the circle $|z| = 2$, described in +ve sense.

8. Find Taylors expansion of the function $f(z) = \frac{z}{z^2 + 9}$ around $z = 0$. Also determine the radius of convergence of the Taylor's series developed.

9. (a) Describe different kinds of singularity.

   (b) What is pole of a function ? Also introduce the residue at simple pole and pole of order m.

10. Evaluate the following integrals :—

    (a) $\int_0^{2\pi} \frac{d\theta}{1 + a\,Cos\theta},\ a^2 < 1$ (b) $\int_0^{\infty} \frac{dx}{(1 + x^2)^2}$

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–VII

(Theory of Differential Equations)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Define Lipschitz condition in a region. Show that the following function do not satisfy the Lipschitz condition in the region indicated : $f(x,\, y) = \dfrac{\sin y}{x},\;\; f(0,\, y) = 0,\;\; |x| \le 1,\;\; |y| < +\propto$.

2. Determine the constants M, C and α for initial value problem $(IVP)\, y' = \sin y,\; y\left(\dfrac{\pi}{2}\right) = 1$, $R = \left\{(x,\, y) : \left|x - \dfrac{\pi}{2}\right| \le \dfrac{\pi}{2},\; |y - 1| \le 1\right\}$ to have solution.

3. (a) Compute the first three successive approximations for solution of the equation $y' = \dfrac{y}{1 + y^2},\; y(0) = 1$.

   (b) Define an $\in$-approximate solution to $y' = f(x,\, y)$ and show its existence.

4. State and prove Picard-Lindelof theorem.

5. Let $A(x)$ be a $n \times n$ matrix with continuous elements on the interval $I : a \le x \le b$. Suppose $B(x)$ is a matrix of functions on I satisfying $B(x) = \lfloor b_{ij}(x) \rfloor \in C'[a,\, b]$. Then show that for $r,\, x \in I$, $\det B(x) = \det B(r)\, \exp \int_r^x tr\, A(t)\, dt$.

6. A necessary and sufficient condition that a solution matrix G be a fundamental matrix, is that $G(x) \ne 0$ for $x \in I$.

7. (a) Find e^A if $A = \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$.

   (b) Solve by matrix method the system of equation $\dfrac{dx_1}{dt} = 9x_1 - 8x_2,\; \dfrac{dx_2}{dt} = 24x_1 - 8x_2$ where $x_1(0) = 1,\;\; x_2(0) = 0$

8. (a) Compute Rodrigue's formula for Legendre polynomial.

   (b) Derive orthogonal properties of Legendre polynomial.

9. Prove the following recurrence relations for Laugerre polynomials.

   (a) $L_n'(x) = -\sum_{r=0}^{n-1} L_r(x)$ (b) $x\, L_n'(x) = nL_n(x) - nL_{n-1}(x)$.

10. (a) Derive the expression for the generating function for Bessel's function.

    (b) Prove that,

    (i) $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer.

    (ii) $J_n(-x) = (-1)^n J_n(x)$, where n is positive or negative integer.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VIII
(Set Theory, Graph Theory, Number Theory, Differential Geometry)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Show that the set Q of all rational numbers is denumerable.
   (b) Prove that $2^{No} = C$.
2. (a) What is Axiom of choice ? Show that the axiom of choice of equivalent to Zermelo's postulates.
   (b) For any three cardinal number $\alpha,\ \beta,\ \gamma$; show that (i) $\alpha^{\beta}\alpha^{\gamma} = \alpha^{\beta+\gamma}$, (ii) $(\alpha\,\beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$.
3. (a) Define isomorphism between two graphs and give two examples of isomorphic graphs.
   (b) Determine the difference between a circuit and Eulerian circuit.
4. (a) Prove that a pseudograph is Eulerian iff it is connected and every vertex is even.
   (b) A complete graph of n vertices is planar if $n \le 4$. Prove this statement.
5. (a) State and prove the division algorithm of integers.
   (b) Define congruency between two integers under a positive integer m. Prove that the relation $a \equiv b$ (mod m) defines an equivalence relation on the set of integers.
6. (a) State and prove Chinese remainder theorem.
   (b) Show that $(a, m_1) = 1,\ (a, m_2) = 1 \Leftrightarrow (a, m_1 m_2) = 1$.
7. (a) State and prove Fermat's theorem.
   (b) If $x \equiv a \pmod 7 \equiv b \pmod{11} \equiv c \pmod{13}$, then prove or disprove that $x \equiv -286a + 364b - 77c \pmod{1001}$.
8. (a) What is a circular helix ? Find the osculating plane at the point $P(\theta)$ on the helix $x = a\,Cos\theta,\ y = a\,Sin\theta,\ z = c\theta$.
   (b) Prove that $\left[\vec{r}^{\,I}, \vec{r}^{\,II}, \vec{r}^{\,III}\right] = \dfrac{T}{\rho^2}$, where $\vec{r}$ the current point, T is torsion and ρ is the radius of curvature.
9. (a) Define Bertrand curves. Prove that the angle between corresponding tangent lines of the two curves is constant.
   (b) Introduce asymptotic line and prove that an asymptotic line is a curve on a surface such that the osculating plane at each point is the tangent plane to the surface at the point.
10. (a) Introduce the concept of developable and ruled surfaces. Prove that the surface $xy = (z - c)^2$ is developable.
    (b) Find the curvature of a normal section of the right helicoid $x = u\,Sin\theta,\ y = u\,Sin\theta$, $z = c\theta$.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Find a relation between difference and differential operator.
   (b) Obtain the missing terms in the following table :—

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
f(x)	0.135	?	0.111	0.100	?	0.082	0.024

2. (a) Find the polynomial of lowest degree which is interpolated by the following sequence of numbers using any method :— 0, 7, 26, 63, 124, 215, 342, 511.
   (b) Define factorial notation and that $(x)^{(-n)} = \frac{1}{(x+hn)^{(n)}}$ where h is the interval of differencing.

3. (a) Find $f(6)$, when $f(0) = 3, f(1) = 6, f(2) = 8, f(3) = 12$ and the third difference being constant.
   (b) Prove that $(1+\Delta)(1-\nabla) = 1$.

4. (a) Find a positive root of $xe^x - 1 = 0$ lying between 0 and 1, using iteration method.
   (b) Compare Newton's method with Regula-Falsi method. Apply Newton's Raphson method to find square root of 12 to five places of decimal.

5. (a) Fit a straight line to the given data regarding x as the independent variable :—

x	1	2	3	4	5	6
y	1200	900	600	200	110	50

   (b) Find the first term of the series whose second and subsequent terms are 8, 3, 0, –1, 0, ...

6. (a) Describe Newton's iterative formula to find a square root and a inverse square root of a number.
   (b) Find real root of the equation $x^2 + 4 Sin\, x = 0$ correct to four places of decimal by using Newton-Raphson method.

7. Form Gauss' central difference table and apply it to determine $e^{1.17}$ from the table :—

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

8. Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2.03$ by Newton's Backward difference formula using the following table :—

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

9. (a) Formulate Quadrature formula for equally spaced arguments and derive Simpson's three-eighth rule.
   (b) Determine the value of the integral $\int_4^{5.2} \log x\, dx$ by applying Trapezoidal rule.

10. (a) Form the difference equation corresponding to the family of curves $y_x = ax^2 + bx - 3$.
    (b) Solve the equation $y_{x+3} + y_{x+2} - y_{x+1} - y_x = 0$, where $y_0 = 2,\ y_1 = -1,\ y_2 = 3$.

* * *

Examination Programme, 2015

M.Sc. Mathematics, Part-II

Date	*Papers*	*Time*	*Examination Centre*
06.06.2015	Paper-IX	3.30 PM to 6.30 PM	Nalanda Open University, Patna
08.06.2015	Paper-X	3.30 PM to 6.30 PM	Nalanda Open University, Patna
10.06.2015	Paper-XI	3.30 PM to 6.30 PM	Nalanda Open University, Patna
12.06.2015	Paper-XII	3.30 PM to 6.30 PM	Nalanda Open University, Patna
16.06.2015	Paper-XIII	3.30 PM to 6.30 PM	Nalanda Open University, Patna
18.06.2015	Paper-XIV	3.30 PM to 6.30 PM	Nalanda Open University, Patna
20.06.2015	Paper-XV	3.30 PM to 6.30 PM	Nalanda Open University, Patna
22.06.2015	Paper-XVI	3.30 PM to 6.30 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–X
(Functional Analysis)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Define a quotient space. Let M be a closed linear sub-space of a normed linear subspace N. If the norm of a coset $x + M$ in the quotient space N/M is defined by $\| x + M \| = \inf \{ \| x + M \| : x \in M \}$, then show that N/M is a normed linear space. Morover in case N also happens to be a Banach space, then show that N/M is also a Banach space.

2. (a) Define normed linear space and Banach space. Prove the following inequality in case of normal linear space. $\left| \| x \| - \| y \| \right| \leq \| x - y \|$.
   (b) If N be a non-zero normed linear space, then show that N is a Banach space iff $\{ x : \| x \| = 1 \}$.

3. If $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, then prove that dual of l_p is l_q.

4. (a) Show that, $x_n \to x$ $w.r.t.$ $\| \quad \|$ if and only if $x_n \to x'$ $w.r.t.$ $\| x \|'$.
   (b) If X and Y be two normed linear spaces and X be finite dimensional, then show that every linear map from X to Y, is continuous.

5. State and prove Hahn-Banach theorem.

6. (a) If the mapping $T \to T^*$ is a norm preserving mapping of $\beta(N)$ to $\beta(N^*)$, then prove that,

   (i) $(\alpha T_1 + \beta T_2)^* = \alpha T_1^* + \beta T_2^*$ and $(T_1 T_2)^* = T_2^* T_1^*$.
   (b) If x and y are Banach spaces and T is a continuous linear transformation of X into Y, then show that T is an open mapping.

7. (a) Sate and prove F. Ries$z$'s lemma.
   (b) Let M be a closed linear sub space of a Hilbert space H and x be a vector not in M, and d be the distance from x to M. Then show that, there exists a unique $y_0 \in M$ such that $\| x - y_0 \| = d$.

8. (a) If M and N are closed linear sub-spaces of a Hilbert space H such that $M \perp N$, then prove that the linear sub space M + N is also closed.
   (b) Let L be a linear space over F. Show that the sum of two inner products on L, is an inner product on L. Is the difference of two inner products is an inner product ? Show that a positive multiple of an inner product is an inner product.

9. (a) State and prove Bessel's inequality for finite orthonormal set.
   (b) Let H be a Hilbert space. Then show that the conjugate space H^* is also a Hilbert space.

10. (a) If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other, then prove that $N_1 + N_2$ and $N_1 N_2$ are normal.
    (b) If a Hilbert space H is separable, then show that every orthonormal set of H is countable.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XI
(Partial Differential Equations)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Explain Charpit's method for the solution of non-linear partial differential equations of first order.
   (b) Apply Charpit's method to find solution of the partial differential equation $\left(p^2 + q^2\right) y = qz$.
2. (a) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and passes through the circle $x^2 + y^2 = 1,\ z = 1$.
   (b) Find the general solution of the partial differential equation $px(x+y) = qy(x+y) - (x-y)(2x+2y+z)$.
3. (a) Explain the computation of particular integral of different cases of non-homogeneous linear partial equations.
   (b) Solve the equation $\left(D - D'^2\right) z = Cos(x - 3y)$.
4. (a) Describe the Jacobi's method for the solution of the partial differential equation $F(x,\ y,\ z,\ p,\ q) = 0$.
   (b) Solve the partial differential equation $xyr + x^2 s - yp = x^3 e^y$.
5. Reduce the equation $yr + (x+y)s + xt = 0$, to canonical form and hence kind its solution.
6. Reduce the partial differential equations (i) $x^2 r + 2xys + y^2 t = 0$ and
   (ii) $t - s + p - q\left(1 + \frac{1}{x}\right) + \frac{z}{s} = 0$, to the canonical forms and hence or otherwise solve them.
7. (a) Show that the family of surfaces defined by $x^2 + y^2 =$ constant, is a family of equi-potential surfaces in free space and then, find the law of potential.
   (b) Solve the boundary value problem $\frac{\partial u}{\partial x} = u\frac{\partial u}{\partial y}$, when $u(0,\ y) = 8e^{-3y}$.
8. (a) Use the method of separation of variables to solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.
   (b) Find $v(x,\ y)$ such that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, satisfies the conditions (i) $V \to 0\ as\ x \to \propto$, (ii) $V(x, 0) = V(x, a) = 0$.
9. (a) Derive the Fourier equation of heat conduction.
   (b) A rod of length *l* with insulated sides, is initially at a uniform temperature u_0. Its ends are suddenly cooled to *0°c* and are kept at that temperature. Find the temperature *u (x, t).*
10. Solve the following boundary value problem : $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2},\ 0 \le x \le \ell, t \ge 0$. Subject to the boundary conditions $\left.\begin{matrix} u(0, t) = 0,\ t > 0 \\ \frac{\partial u}{\partial x}(\ell, t) = 0, t > 0 \end{matrix}\right\}$ and initial conditions $u(x, 0) = \begin{cases} x,\ 0 \le x < \frac{\ell}{4} \\ \frac{\ell}{2} - x, \frac{\ell}{4} \le x < \frac{\ell}{2} \\ 0,\ \frac{\ell}{2} \le x < \ell \end{cases}$ and $\frac{\partial u}{\partial t}(x, 0) = 0, 0 \le x < \ell$.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XII
(Analytical Dynamics)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Derive the formula for kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of kinetic energy.
   (b) Express the difference between possible displacement and virtual displacement. Give examples of these kinds of displacements.

2. (a) Derive Lagrange's equation of motion from Hamilton's equation (canonical equations).
   (b) Apply Lagrange's equations of motion to find the equations of motion of a simple pendulum.

3. (a) In a dynamical system, if the time of passing from one configuration to another is prescribed, then prove that Hamilton function has a stationary value along the actual path.
   (b) State Hamilton's principle and construct the equation of motion of one dimensional harmonic oscillator.

4. What is Hamilton's function ? Find the differential equations for Hamilton's function.

5. Discuss the motion of spherical pendulum deducing from Hamilton's canonical equations of motion.

6. What is small oscillation of a dynamical system ? Describe Lagrange method of solution of small oscillation and compare it with the approximation of the potential and kinetic energy of the dynamical system.

7. (a) Give idea of canonical transformation and state conditions for a transformation becoming canonical.
   (b) Define Lagrange's bracket. Prove that Lagrange's bracket is invariant under canonical transformation.

8. (a) Show that Lagrange Bracket does not obey the commutative law of algebra.
   (b) Prove that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical. Find the generating function *F(q, Q)*.

9. (a) Derive Hamilton-Jacobi equation and then Hamilton's characteristic function.
   (b) Give the physical significance of Hamilton characteristic function.

10. A small sphere rolls without slipping on the rough interior of a fixed vertical cylinder of greater radius. Discuss the motion of sphere.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIII
(Fluid Mechanics)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Explain Lagrangian and Eulerian method of study.

   (b) What is summation due to Einstein ? How this helps the study of big expressions ?

2. (a) Derive the equation of Continuity in Cartesian form.

   (b) Write short notes on (i) Velocity Potential, (ii) Velocity Vector and (iii) Boundary Surface.

3. (a) The velocity $\vec{q}$ in a three dimensional flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}$. Determine the equations of streams passing through the point (1, 1, 1).

   (b) Prove that liquid motion is possible when velocity at (x, y, z) is given by $u = \frac{3x^2 - r^2}{r^5}$, $v = \frac{3xy}{r^5}$, $w = \frac{3xz}{r^5}$.

4. (a) Derive the equation of motion under impulsive force (in vector form).

   (b) A sphere of radius 'a' surrounded by infinite liquid of density ρ, the pressure at infinity being π. The sphere is suddenly annihilated. Show that the pressure at a distance *r* from the centre immediately falls to $\pi\left(1 - \frac{a}{r}\right)$.

5. (a) A velocity field is given by $\vec{q} = -x\hat{i} + (y+t)\hat{j}$. Find the stream function and the stream lines for field at *t = 2*.

   (b) Define Source, Sink and Doublet and give their suitable examples.

6. (a) Derive Euler's equation of motion in cylindrical polar co-ordinates.

   (b) A velocity field is given by $\vec{q} = \frac{-iy + \hat{j}x}{x^2 + y^2}$ calculate the circulation round the square having corners at (1, 0), (2, 0), (2, 1) and (1, 1). Also test the flow for rotation.

7. (a) Describe the motion of a fluid between rotating co-axial circular cylinders.

   (b) A circular cylinder is fixed across a stream of velocity *U* with circulation *K* round the cylinder. Show that the maximum velocity in the fluid is $2U + \frac{K}{2\pi a}$, where *a* is the radius of fixed cylinder.

8. Show that a sphere projected in a liquid under gravity, describes a parabola of the latus rectum $\frac{2\sigma + \rho}{\sigma - \rho}\frac{W}{g}$, where σ and ρ are the densities of the sphere and the liquid respectively, and *W* is the horizontal velocity of the sphere.

9. (a) Derive the rate of strain tensor of fluid in motion.

   (b) Show that the velocity field defined at a point *P* by $(1 + 2y - 3z, 4 - 2x + 5z, 6 + 3x - 5y)$ represents a rigid body rotation.

10. (a) Describe the dissipation of energy due to viscousity.

    (b) Incompressible liquid is flowing steadily through a circular pipe. Prove that the mean pressure is constant over the cross-section and the rate of flow is $\frac{\pi a^4 (p_1 - p_2)}{8\mu\ell}$, where p_1 and p_2 are the pressures over the sections a distance ℓ apart.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIV
(Operations Research)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a convex set in R^n. Show that the set $S = \{(x_1, x_2) : x_1^2 + x_2^2 \le 1, x_1 + x_2 \ge 1\}$ is a convex set in R^2.
   (b) Give the notion of Basic Feasible and Non-Basic Feasible solutions. Find all basic feasible solutions of the system $x_1 + 2x_2 + 4x_3 + x_4 = 7,\ 2x_1 - x_2 + 3x_3 + 2x_4 = 4$ and testify for degenerate solution.
2. (a) Solve graphically the L.P.P. (if possible)
   Solve the following L.P.P. by graphical method (if solvable);
   Minimize $z = 4x_1 + 7x_2$
   Subject to $x_1 + 2x_2 \le 20,\ x_1 + x_2 \le 15,\ x_2 \le 8$ *and* $x_1 \ge 0,\ x_2 \ge 0$
   (b) The system of equations $x_1 + 2x_2 + 4x_3 + x_4 = 7$ and $2x_1 - x_2 + 3x_3 + 2x_4 = 4$ has a feasible solution $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 0$. Convert this feasible solution to a basic feasible solution.
3. Solve the following L.P.P. by simplex method
   Maximize $z = 4x_1 + 10x_2$
   Subject to $2x_1 + x_2 \le 50,\ 2x_1 + 5x_2 \le 100,\ 2x_1 + 3x_2 \le 90;\quad x_1 \ge 0, x_2 \ge 0$.
4. Apply two phase simplex method to solve the following L.P.P.
   Minimize $z = \frac{15}{2}x_1 - 3x_2$
   Subject to $3x_1 - x_2 - x_3 \ge 3,\ x_1 - x_2 + x_3 \ge 2,$ *where* $x_i \ge 0\ (i = 1, 2, 3)$.
5. By elaborating each step of the Dual Simplex Method, solve the following L.P.P.
   Maximize $z = -3x_1 + x_2$
   Subject to $x_1 + x_2 \ge 1,\ 2x_1 + 3x_2 \ge 2$ *and* $x_1 \ge 0, x_2 \ge 0$.
6. State and prove Basic Duality theorem.
7. Consider the L.P.P.
   Maximize $z = -x_1 + 2x_2 - x_3$
   Subject to $3x_1 + x_2 - x_3 \le 10,\ -x_1 + 4x_2 + x_3 \ge 6,\ x_2 + x_3 \le 4$ *and* $x_j \ge 0\ (j = 1, 2, 3)$.
   Determine the ranges for discrete changes in the components b_2 and b_3 of the requirement vector $\vec{b}$ so as to maintain the feasibility of the current optimum solution.
8. (a) Describe various steps involved in the formulation of a primal problem to its dual.
   (b) Obtain the dual of the following L.P.P. :—
   Maximize $z = 2x_1 + 3x_2 + x_3$
   Subject to $4x_1 + 3x_2 + x_3 = 6,\ x_1 + 2x_2 + 5x_3 \le 4$ where $x_1 \ge 0,\ x_2 \ge 0$ and x_3 is unrestricted.
9. Solve the following game problem :—

   Player B

   $$\text{Player } A \begin{bmatrix} 4 & 2 & 4 \\ 2 & 4 & 0 \\ 4 & 0 & 8 \end{bmatrix}$$
10. Solve the following non-linear problem graphically :—
   Maximize $z = 8x_1 - x_1^2 + 8x_2 - x_2^2$
   Subject to $x_1 + x_2 \le 12,\ x_1 - x_2 \ge 14$ *and* $x_1 \ge 0,\ x_2 \ge 0$ (Draw the graph neat and clean).

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XV
(Tensor Algebra, Integral Transforms, Integral Equations, Operational Research Modeling)

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a relative tensor of weight w. Prove that equations of transformation of a relative tensor possess group property.

   (b) If the tensors (a_{ij}) and (b_{ij}) are symmetric and u^i, v^j are components of contravariant vectors satisfying the equations $\left.\begin{matrix}(a_{ij} - k\, b_{ij})\, u^i = 0 \\ (a_{ij} - k' b_{ij}) v^i = 0\end{matrix}\right\} i, j = 1, 2, ----, n \;\; and \;\; k \neq k'$.

   Show that $a_{ij}\, u^i\, v^j = 0 \;\; and \;\; b_{ij}\, v^j = 0$.

2. (a) Prove that the outer product of two tensors *(r, s)* and *(p, q)* types is a tensor of the *(r + p, s + q)* type.

   (b) Define a mixed tensor. Show that by each process of contraction of a mixed tensor, the rank of the tensor is reduced by two.

3. (a) Show that the covariant derivative of a contravariant vector, is a mixed tensor of rank two.

   (b) What are Christoffel symbols of first and second kind ? Determine the transformation law of Christoffel symbol of first kind.

4. (a) Define Laplace transform and prove its linearity property.

   (b) If *f(s)* be the fourier transform of *F(x)*, then show that $\frac{1}{a} f\left(\frac{s}{a}\right)$ is the Fourier transform of *F(ax)*.

5. State convolution theorem and apply it to find

   (a) $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$ (b) $L^{-1}\left\{\frac{s}{s^2 + a^2}\right\}$

6. (a) Explain the method of construction of solution for Fredholm Integral Equation.

   (b) Form an integral equation corresponding to the differential equation $\frac{dy}{dx^2} - Sin\, x \frac{dy}{dx} + e^x y = x$ with the initial conditions $y(0) = 1$ and $y'(0) = -1$.

7. (a) Describe Fredholm integral and Volterra integral equations.

   (b) Prove that the function $u(t) + (1 + x^2)^{-1/2}$, is a solution of the Volterra integral equation $u(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} u(t) dt$.

8. Explain about the solution of Fredholm integral equation having separable kernel of second kind.

9. Describe deterministic model with instantaneous production (shortages allowed).

10. The maintenance cost and resale value per year of a machine whose purchase price is Rs. 7,000/– is given below :—

Year	1	2	3	4	5	6	7	8
Maintenance Cost (Rs.)	900	1200	1600	2100	2800	3700	4700	5900
Resale Value (Rs.)	4000	2000	1200	600	50	400	400	400

When should the machine be replaced ?

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XVI
(Programming in 'C')

Annual Examination, 2015

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. How string variables declared and initialized ? Explain giving a proper example.
2. (a) Write a programme to check whether number entered from the keyboard on user's choice is prime number.
   (b) Input a number and check whether it is even or odd.
3. Explain Relational Operator, Conditional Operator and Evaluation Operator with examples.
4. What is an expression ? What are its components ?
5. What is an array ? Write a programme to input and print all the elements of 3 × 3 matrix.
6. Describe two different ways to utilize the increment and decrement operators ? How do the two methods differ ?
7. What are structures in C and why are they used. How are they initialized ? Explain the purpose of type def feature with an example.
8. Explain types of looping statements with examples. What is difference between while loop and do-while loop ?
9. Explain the concept of pointers in C with suitable examples. How can function return a pointer to its calling routine ?
10. What is function ? Are functions required when writing a C program ?

* * *

M.Sc. Mathematics, Part–II, Paper–XVI (Practical)
Counselling & Examination Programme, 2015

Practical Counselling Programme

Enrollment No.	*Date*	*Time*	*Venue*
All Old & New Students	23.06.2015 to 29.06.2015	1:00 PM to 5:00 PM	Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001

Practical Examination Programme

Enrollment No.	*Date*	*Time*	*Venue*
All Old & New Students	30.06.2015	12.00 PM to 3.00 PM	Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001