NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–I
(Advanced Abstract Algebra)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define sub-normal series of a group giving an example of it.
   (b) Present two isomorphic refinements of a given sub-normal series $Z \supset 20Z \supset 60Z \supset \{0\}$ and $Z \supset 49Z \supset 245Z \supset \{0\}$.

2. State and prove Zassenhaus Butterfly Lemma.

3. (a) By giving an idea of units, find all the units of the integral domain of Gaussian Integers.
   (b) In an integral domain D with unity, every prime element is irreducible but its converse is not necessarily true. Prove this statement.

4. (a) After defining Euclidean ring, prove that every Euclidean ring is a principal ideal ring.
   (b) In Z [x] the ring of polynomials over Z, find the H.C.F. and L.C.M. of $x^2 + 2x$ and $2x + 4$.

5. State the general properties of modules and hence prove them.

6. Define kernel of homomorphism from a module *M* into a module *N′*. Prove that if *f* is a module homomorphism, then *f* is an isomorphism iff $K(f) = \{0\}$.

7. Establish the transitivity property of finite extension of fields.

8. (a) Let $a \in K$ be an algebraic over a field *F* and $p(x)$ be a minimal polynomial over *F.* Then, show that *p(x)* is irreducible over *F*.
   (b) Show that any finite extension of a field of characteristic zero, is a simple extension.

9. (a) Define fixed field of a sub-group *A* of the group *G* of all automorphisms of a field *F.* Show that the fixed field *A* is a sub-field of *F.*
   (b) Let *K* be a finite separable normal extension of a field *F* of characteristic 0, then show that the fixed field of Galois group *G[K : F]* is *F* itself.

10. If *K* be an extension of the field *Q* of rationals, then show that any automorphism must leave every element of *Q* fixed.

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Examination Programme, 2013
M.Sc. Mathematics, Part–I

Date	*Paper*	*Time*	*Examination Centre*
17.07.2013	Paper–I	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna
19.07.2013	Paper–II	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna
23.07.2013	Paper–III	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna
25.07.2013	Paper–IV	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna
27.07.2013	Paper–V	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna
29.07.2013	Paper–VI	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna
31.07.2013	Paper–VII	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna
02.08.2013	Paper–VIII	3.30 PM to 6.30 PM	D.A.V. Public School Punaichak, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–II
(Real Analysis)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) State and prove Cantor's intersection theorem.
   (b) Show by giving counter-example that Heine-Borel theorem can't be extended to unbounded intervals.
2. Show that a function f defined on *[a, b]* is of bounded variation iff it can be represented as difference of two monotonically increasing functions on *[a, b]*.
3. (a) State and prove a necessary and sufficient condition for $f \in R(\alpha)$ on *[a, b]*.
   (b) Let $f_1, f_2 \in R(\alpha)$ on *[a, b]*. Then prove that $f_1 + f_2 \in R(\alpha)$ and $cf_1, cf_2 \in R(\alpha)$ for every constant c and $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$ and $\int_a^b (cf_1)\, d\alpha = c\int_a^b f_1\, d\alpha$.
4. (a) If $f \in R(\alpha)$ and α is monotonically increasing on *[a, b]*, then show that $|f| \in R(\alpha)$ on *[a, b]* and $\left|\int_a^b f\, d\alpha\right| \le \int_a^b |f|\, d\alpha$.
   (b) Let f be an arbitrary function and α be a constant function on *[a, b]*. Prove that $f \in R(\alpha)$ on *[a, b]* and $\int_a^b f\, d\alpha = 0$.
5. (a) Let f be a vector-valued function defined on an sub-set E of R^n with values in R^m. If both partial derivatives $D_i \underline{f}$ and $D_j \underline{f}$ exist in an n-ball $B(C, \delta)$ and if both are differentiable at a point $\underline{e}$ of E, then prove that $D_{i,j} f(c) = D_{j,i} f(c)$.
   (b) Let $f : R^2 \to R$ be defined by $f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}$ when $(x, y) \ne (0, 0)$ and $f(0, 0) = 0$, then show that $D_{12} f(0, 0) \ne D_{21} f(0, 0)$.
6. Let $T : R^n \to R^m$ be a vector valued linear mapping. Prove that there exists a $m \times n$ matrix corresponding to T and conversely given a matrix of order $m \times n$, there corresponds a linear function of R^n to R^m.
7. (a) If a power series $\sum a_n x^n$ converges to a function $f(x)$ for $|x| < r$. Then f is continuous and differentiable in $]-r, r[$ such that $f'(x) = \sum n a_n x^{n-1}$, for $|x| < r$.
   (b) Define Abel's summability of a series. Prove that the series $\sum (-1)^n$ is Abel summable to ½.
8. Define maximum and minimum values of a real valued function $f(x, y)$ of two real variables and show that the necessary conditions for f to have an extreme value $f(a, b)$ are that $f_x(a, b) = 0 = f_y(a, b)$, where f_x and f_y are partial derivatives of f. Show further that these conditions are not sufficient.
9. State and prove the Inverse Function theorem.
10. (a) Define functional dependence on a set $E \subset R^n$. Show that the functions $f_1(x, y) = Cos(x + y)$, $f_2(x, y) = Sin(x + y)$ are functionally dependent on R^2 with the dependence function ϕ.
    (b) Show that the zeros of any real-valued function which is continuous on R^n form a closed set and conversely, if C is a closed set in R^n, then there exists a continuous function ϕ on R^n such that $\phi(x) = 0$ iff $x \in C$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–III
(Measure Theory)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define outer measure of a set. Prove that an outer measure is a non-decreasing function over a set.
   (b) If E be a given set and $\in > 0$, then there exists an open set $0 \supset E$ such that $m^*(0) < m^*(E) + \in$. Prove this statement.
2. (a) If E_1 *and* E_2 are disjoint sets, then prove that $m^*(E_1 \cup E_2) = m^*(E_1) + m^*(E_2)$.
   (b) Prove that the complement of every measurable set *E*, is also measurable.
3. (a) If *A* and *B* are measurable sets, then show that $m(A) + m(B) = m(A \cup B) + m(A \cap B)$.
   (b) Define a measurable space. Give an example of a measurable space. Give an example of a measurable space and its measure (with justification).
4. (a) By introducing the characteristic function of a set *A*, prove that the characteristic function of *A* is measurable iff *A* is measurable.
   (b) Prove that a step function is measurable.
5. (a) Prove that a necessary and sufficient condition for a function *f* to be measurable is that it is the limit of convergence of simple functions.
   (b) Examine the measurability of the function *f* given below : $f(x) = \begin{cases} x+5, \text{ if } x < -1 \\ 2, \text{ if } -1 \le x < 0 \\ x^2, \text{ if } 0 \le x \end{cases}$.
6. (a) Show that every function *f* which is *R*-integrable on *[a, b]* is also *L*-integrable on *[a, b]* and $(R)\int_a^b f(x)dx = (L)\int_a^b f(x)dx$.
   (b) If *f(x) = 0* at every point of cantor's discontinuous domain and *f(x) = p* in each of the complementary intervals of length $3^{-p}(p = 1, 2, 3, ---)$, show that $\int_0^1 f\,dx$ exists in Lebesgue sense and is equal to 3.
7. (a) State and prove Lebesgue Monotone convergent theorem.
   (b) Derive that the Lebesgue integral is σ-additive on the family of non-negative measurable functions.
8. Let (u_n) be a sequence of integrable functions on E such that $\sum_{n=1}^{\infty} u_n$ converges almost everywhere on *E*. Let *g* be a function which is integrable on *E* and satisfies the relation $\left|\sum_{i=1}^{n} u_i\right| \le g$ almost everywhere on *E* for each *n*, then show that $\int_E \sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \int_E u_n$.
9. State and prove Jordan Decomposition theorem.
10. (a) Show that an absolutely continuous functions is continuous, but the converse is not true.
    (b) if *f* is integrable over *[a, b]* and $\int_a^x f(t)dt = 0$ for all *x* in *[a, b]*, then prove that *f(x) = 0* almost everywhere in *[a, b]*.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–IV
(Topology)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) If $(T_i)_{i \in I}$ be a family of topologies on a non-empty set *x*. Then prove that $\bigcap_{i \in I} T_i$ is also a topology on *x*.
   (b) Show that the boundary of a closed set is no-where dense.

2. Let *A* be a sub-set of a topological space *(X,T)* then show that (i) $(Int\ A)' = \overline{A'}$, (ii) $(\overline{A})' = Int(A')$, where dash denotes the complement of the set.

3. (a) In a Hausdorff space, every convergent sequence has a unique limit.
   (b) Show that a topological space *x* is a T_0-space iff $x,\ y \in x\ \&\ x \neq y \Rightarrow \overline{\{x\}} \neq \overline{\{y\}}$.

4. Prove that a necessary and sufficient condition for a one-to-one mapping $f : x \to y$ to be a homeomorphism is that $f(\overline{A}) = \overline{f(A)}$ for every $A \subseteq X$ (*X* and *Y* being topological spaces).

5. (a) Give an example (with proof) of a topological space which is a T_1-space but not a T_2-space.
   (b) Show that a finite sub-set of T_1-space has no cluster point.

6. (a) Define a normal space. Give an example of a normal space which is not regular. Justify this statement with relevant logic.
   (b) Show that every metric space is a normal space.

7. (a) Show that the open interval *(0, 1)* on the real line *R* is not compact.
   (b) Prove that every compact sub-space of a Housdorff space is closed.

8. (a) Define a T_3-space and T_4-space. Prove that every T_4-space is a T_3-space.
   (b) Let *B* be a base for a topology on *x*. Then prove that *x* is compact iff every basic open cover has a finite sub-cover.

9. (a) Let *x* be a topological space. Prove that *x* is disconnected iff there exists a non-empty proper sub-set of *x* which is both open and closed.
   (b) Show that connectedness is not a hereditary property.

10. If *x* and *y* are topological spaces, then show that *x × y* is connected iff *x* and *y* are connected.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–V

(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that a linear operator E is projection on some sub-space iff it is an idempotent.
   (b) If E is a projection on M along N, then show that $I–E$ is a projection on N along M and vice-versa, where V is direct sum of sub-spaces M and N.

2. (a) Let $V(F)$ be a finite dimensional vector space and W is a sub-space of V, then prove that $\dim V/W = \dim V - \dim W$.
   (b) Define isomorphism between two vector spaces. Let $f: R^2 \to R^2$ defined by $f(x, y) = (y, x)$, then, show that f is an isomorphism.

3. (a) What do you understand about a bilinear form ? If f be the scalar product on R^n given by $f(u, v) = u.v = a_1b_1 + a_2b_2 + ---- + a_nb_n$, where $u = (a_1, a_2, ----, a_n)$ and $v = (b_1, b_2, ----, b_n)$. Then, prove that f is a bilinear form on R^n.
   (b) Show that a bilinear form f on $V(K)$ is skew-symmetric iff the matrix A of this bilinear form in some ordered basis, is skew-symmetric.

4. (a) Show that a Hermitian form remians Hermitian by a non-singular transformation.
   (b) Let $\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$ be a basis of Euclidean space R^3. Find its orthonormal basis.

5. (a) Define a partially ordered set. Prove that the power set of a non-empty set x forms a partially ordered set with respect to sub-set relation $(\subseteq)$.
   (b) Introduce complete lattice and show that the lattice $(N, \subseteq)$ is incomplete lattice.

6. (a) Establish the equivalence of two distributive laws :—
   (i) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ (ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
   (b) Prove that 0 and 1 are fixed points for an order-homomorphism on a complete lattice.

7. (a) Prove the idempotent laws : (i) $x \vee x = x$, (ii) $x \wedge x = x$, for all $x \in B$ (Boolean Algebra).
   (b) Verify the validity of De' Morgan laws in Boolean Algebra : (i) $(x \vee y)' = x' \wedge y'$, (ii) $(x \wedge y)' = x' \vee y'$.

8. (a) Prove that a Boolean homomorphism is a monomorphism iff $Ker(f) = \{0\}$.
   (b) Prove that the kernel of Boolean homomorphism is a proper ideal.

9. (a) Give two examples of linear operators on a vector space $V(K)$.
   Show that, (i) $\det(T_1 T_2) = (\det T_1)(\det T_2)$ and (ii) T is invertible iff $\det T \neq 0$.
   (b) Reduce $A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ to a triangular form.

10. (a) Define in variant sub-space under a linear operator and give a suitable example of it.
    (b) Convert $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$ to Jordan canonical form.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–VI
(Complex Analysis)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Derive Cauchy's Riemann Differential Equations for an analytic function in cartesian form.
   (b) Find the analytic function $f(z)$ of which the real part $u(x, y) = e^x(x\,Cos\,y - y\,Sin\,y)$.

2. (a) Define circle of convergence and prove that the sum function of the power series $\sum a_n z^n$ becomes an analytic function inside the circle of convergence.
   (b) Find the domain of convergence for the series $\sum_{n=1}^{\infty}\left(\frac{iz-1}{2+i}\right)^n$.

3. By introducing Bilinear Transformation, derive the existence of fixed points of a Bilinear Transformation.

4. (a) State and prove the necessary and sufficient condition for the transformation $w = f(z)$ to be conformal.
   (b) Show that the transformation $w \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the w–plane and hence find its centre.

5. (a) If $f(z)$ is analytic function of z and is continuous at each point within and on a closed contour C, then prove that $\int_C f(z)dz = 0$.
   (b) Show that $e^{C/2\left(z - \frac{1}{z}\right)} = \sum_{-\infty}^{\infty} a_n z^n$, where $a_n = \frac{1}{2\pi}\int_0^{2\pi} Cos(n\theta - C\sin\theta)d\theta$.

6. (a) State and prove Taylor's theorem for an analytic function.
   (b) Evaluate $\int_C \frac{dz}{z^2 + 2z + 2}$, where c is the square having vertices at *(0, 0), (–2, 0), (–2, –2)* and *(0, –2)* oriented in anticlock-wise direction.

7. (a) State Poisson's Integral formula and then prove it.
   (b) Evaluate $\int_C \bar{z}\,dz$, where c is the straight line segment from *(1, 0)* to *(1, 1)*.

8. (a) Define singularities and describe different kinds of singularities.
   (b) Use Cauchy's residue theorem to evaluate $\int_0^a \frac{dx}{(1+x^2)^2}$.

9. (a) Calculate the residues of the function $\frac{z+1}{z^2(z-3)}$.
   (b) Introduce Green's function and describe its existence and uniqueness for the related problem.

10. Discuss Neumann's problem and its solution.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–VII

(Theory of Differential Equations)

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Find an interval *I* containing *r* and a solution *g* of $y' = \frac{dy}{dx} = f(x, y)$ on *I* satisfying *g(r) = s*.

   (b) Compute the first three successive approximations for the solution of the equation $y' = y^2, \; y(0) = 1$.

2. State and prove Picard-Lindelof theorem.

3. (a) Determine the constants *M, C* and *x* for the initial value problem $y' = y, \; y(0) = 1$, $R = \{(x, y); |x| \le 1 \text{ and } |y - 1| \le 1\}$.

   (b) Solve the system of differential equations $y_1^1 = y_1^2, \;\; y_2^1 = y_1 + y_2$.

4. (a) Prove that the set of all solutions of linear homogeneous equations on *I* forms a complex vector space of dimension *n,* where *n* is the order of linear homogeneous system.

   (b) Find e^A if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.

5. (a) Define Wronskian of linear differential equations. If $g_1(x), g_2(x), ---, g_n(x)$ are linearly independent solutions of a $L_n(y) = 0$ on some interval *I*, then show that Wronskian $W(g_1, g_2, ---, g_n)(x) \ne 0$ for any $x \in I$.

   (b) Consider the system $y' = Ay$, given that $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, then show that fundamental matrix $G(x) = \begin{bmatrix} e^x & 0 \\ 0 & e^{2x} \end{bmatrix}$.

6. Introduce the concept of phase plane, path and autonomous system with reference to the system of the form $\frac{dx}{dt} = P(x, y)$ and $\frac{dy}{dt} = Q(x, y)$. Hence, determine the above three objects for the system $\frac{dx}{dt} = y, \; \frac{dx}{dt} = -x$.

7. (a) Explain different types of critical points for a system and give the geometrical meaning of each critical point.

   (b) Find the nature of critical point (0, 0) of the system $\frac{dx}{dt} = x + 5y, \; \frac{dy}{dt} = 3x + y$ and discuss their stability.

8. (a) Explain the nature of critical point of a non-linear system $\frac{dx}{dt} = ax + by + \phi(x, y)$, $\frac{dy}{dt} = cx + dy + \psi(x, y)$.

   (b) Determine the type and stability of the critical point (0, 0) of the non-linear system $\frac{dx}{dt} = Sinx - 4y, \; \frac{dy}{dt} = Sin2x - 5y$.

9. (a) Derive the Rodrigue's formula for the Legendre Polynomial $P_n(x)$.

   (b) Find a recurrence relation for the Laguerre Polynomials $L_n^1(x) - nL_{n-1}^1(x) + nL_{n-1}(x) = 0$.

10. (a) Prove that $e^{\frac{x}{2}\left(z - \frac{1}{z}\right)}$ is the generating function for Bessel's function $J_n(x)$ i.e. $e^{\frac{x}{2}\left(z - \frac{1}{z}\right)} = \sum_{n=-\infty}^{\infty} z^n J_n(x)$.

    (b) Formulate the following relations :—

    (i) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$ (ii) $J_{n-1}(x) - J_{n+1}(x) = 2J_n^1(x)$

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–VIII

(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that $2^{N_0} = C$, where N_0 is the cardinal number of the set *N*, where as *C* is the cardinal number of [0, 1].
   (b) State and prove Schroder-Bernstein theorem.

2. (a) For any three cardinal numbers α, β and γ, show that (i) $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$ and (ii) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$.
   (b) If *B* is an infinite sub-set of a denumerable set *A*, then show that *B* is also denumerable.

3. (a) show that a pseudograph is Eulerian iff it is connected and every vertex is even.
   (b) Define isomorphism of two graphs. Give examples of two graphs becoming isomorphic.

4. (a) If *g* is a connected graph (planar) with *e*-edges and *v*-vertices, then prove that $e \le 3v - 6$.
   (b) Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph.

5. (a) Satisfy yourself that for a fixed integer $m \ge 0$, the relation $a \equiv b$ (mod *m*) forms an equivalence relation on the set of integers *Z*.
   (b) Show that $(a, m_1) = 1$ and $(a, m_2) = 1$ iff $(a, m_1 m_2) = 1$, where (a, b) means that *a* & *b* are prime to each other.

6. What do you mean by Euler's ϕ function ? If m_1, m_2 be positive integers such that $(m_1, m_2) = 1$, then show that $\phi(m_1, m_2) = \phi(m_1)\,\phi(m_2)$.

7. (a) State and prove Fermat's theorem.
   (b) Test the solvability of the congruence relation $x^2 = -10(\text{mod}u\ 127)$.

8. (a) State and prove Serret-Frenet formulae.
   (b) Find the radius of curvature and torsion of the helix $x = a\,Cos\,\theta,\ y = a\,Sin\,\theta,\ z = a\,\theta\tan\alpha$.

9. (a) Find the differential equation of the lines of curvature on a surface.
   (b) Show that the lines of curvature of the paraboloid $xy = az$ *lie* on the surface $Sinh^{-1}\frac{x}{a} \pm Sinh^{-1}\frac{y}{a} =$ *constant.*

10. (a) Define a geodesic on a surface and determine its torsion.
    (b) show that the asymptotic lines on the paraboloid $2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ are $\frac{x}{a} \pm \frac{y}{b} = constant$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–IX
(Numerical Analysis)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Find the polynomial of lowest degree which is interpolated by the following sequence of numbers using any methods.
0, 7, 26, 63, 124, 215, 342, 511

(b) Define factorial notation and prove that $x^{(-n)} = \frac{1}{(x+hn)^{(n)}}$ where h is the interval of differencing.

2. (a) Find $f(6)$, when $f(0) = 3, f(1) = 6, f(2) = 8, f(3) = 12$ and the third difference being constant.

(b) Prove that $(1+\Delta)(1-\nabla) = 1$.

3. (a) State and explain Newton-Raphson method.

(b) Obtain square root of 12 to five places of decimal by Newton's Raphson method.

4. (a) Explain Regula-Falsi method geometrically.

(b) Solve $x = 0.21 \, Sin \, (.5 + x)$ by using iteration method beginning with $x = 0.12$.

5. (a) State and prove Lagrange's Interpolation Formula and also find error in this formula.

(b) Find the form of the function given by,

x	3	2	1	–1
f(x)	3	12	15	–21

6. Introduce the concept of divided differences and prove that the value of the divided difference is independent of the order of the argument. Also prove that the *nth* divided differences of a polynomial of the *nth* degree, is constant.

7. (a) Formulate the computation of derivative using Newton's Backward difference formula.

(b) From the following table, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2.03$.

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

8. (a) Derive Simpson's three-eight's rule for the numerical integration of a function.

(b) A river is 80 ft. wide. The depth d (in feet) of the river at a distance x from one bank, is given by the following table :—

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find the area of Cross section using Simpson's $\frac{1}{3}$ rule.

9. (a) Introduce the notion of a difference equation. Form the difference equation of $f(x) = y_x = C_1 3^x + C_2 (-2)^x$, where C_1 and C_2 are constant parameters.

(b) Solve $2y_{x+1} = y_x + 4, \; y_0 = 3$.

10. (a) Describe least squares method for curve fitting of *nth* degree.

(b) Fit a second degree parabola $y = a + bx + cx^2$ to the following population data of city.

Year	1979	1980	1981	1982	1983	1984	1985	1986	1987
Population in Laks	5	6	6	7	7	8	9	10	10

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–X

(Functional Analysis)

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Introduce a Banach space. Give examples of it with full explanation.
   (b) Let x be a non-empty set and M is the set of all bounded real valued functions defined on X. For $f, g \in M$, let $\| f - g \| = \underset{x \in X}{Sup} | f(x) - g(x) |$, then show that $(M, \| \quad \|)$ is a Banach space.

2. Define continuity of linear transformation of a normed linear space to other normed linear space. Prove that for a linear transformation T, $\| T \| = Inf \ \{ k : k \geq 0 \ and \ \| T(x) \| \leq k \| x \| \ for \ all \ x \}$.

3. Give the idea of dual space of normed linear space. Prove that $C^* = l_1$, where C is the Banach space of all convergent sequences of scalars.

4. State and prove Banach-SteinRans theorem.

5. (a) Let L_1 and L_2 be sub-spaces of a normed linear space N, where L_1 is closed and L_2 is finite dimensional, then, show that $L_1 + L_2$ is closed in N.
   (b) Let X and Y be two normed linear spaces and X be finite dimensional. Prove that every linear map of X to Y, is continuous.

6. (a) Define equivalence of norms defined into a normed linear space in two ways. Prove that $x_n \to x$ with respect to norm $\| \quad \|$ iff $x_n \to x$ with respect to the norm $\| \quad \|'$.
   (b) Let M be a closed linear sub-space of a normed linear space N and a be an element not in M. If d is the distance from a to M, then show that there exists a functional f in N^* such that $f(M) = 0, f(a) = 1$ and $\| f \| = \frac{1}{d}$.

7. (a) Let L be an inner product space over a field F. Define a norm on L by $\| x \| = +\sqrt{(x_1 \, x)}$ for all $x \in L$. Then, prove that L is a normed linear space.
   (b) State and prove Cauchy-Schwarz inequality.

8. Let L be a linear space over F. Show that the sum of two inner products on L is an inner product on L. Is the difference two inner products an inner product ? Show that a positive multiple of an inner product is an inner product.

9. (a) Define orthogonal complement of a set S and denote it by $S^{\perp}$. Then prove that $S \subseteq S^{\perp \perp}$.
   (b) If H be a Hilbert space and $\{e_1, e_2, ---, e_n\}$ be an orthonormal set in H. Then show that $\| x \|^2 = \sum_{i=1}^{n} |(x, e_i)|^2$, for all $x \in H$.

10. (a) Define conjugate space of a Hilbert space H and denote by H*. Show that H* is also a Hilbert space.
    (b) Introduce the adjoint of an operator T which is represented by T^*. Then, prove that, (i) $(T_1 + T_2)^* = T_1^* + T_2^*$ and (ii) $(T_1 T_2)^* = T_2^* T_1^*$

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XI

(Partial Differential Equations)

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Find the compatibility of two first order partial differential equations admitting solution.
   (b) Show that the equations $xp - yq = 0$ and $z(xp + yq) = 2xy$ are compatible and hence solve them.

2. (a) Describe Jacobi's method for the solution of the partial differential equation $F(x, y, z, p, q) = 0$.
   (b) Use Jacobi's method to solve the equation $p^2 x + q^2 y = z$.

3. Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$, orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

4. Solve the equations, (i) $(D^2 - 2DD^1)z = e^{2x} + x^2 y$, (ii) $(D^3 - 4D^2D^1 + 4DD^{12})z = Cos(2x + y)$.

5. (a) Introduce the concept of characteristic equation and characteristic curve of a partial differential equation.
   (b) Solve the equation $xr + p = qx^2 y^2$.

6. Reduce the differential equations, (i) $t - s + p - q\left(1 + \frac{1}{x}\right) + \frac{z}{x} = 0$ and (ii) $x^2 r + 2xys + y^2 t = 0$ to the canonical forms and hence or otherwise solve these.

7. Apply variable-separable method to solve the boundary value problem $c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$ satisfying the initial conditions $y\,(x, 0) = f(x),\ 0 \le x < l$ and $\frac{\partial y}{\partial t}(x, 0) = g(x), 0 \le x < l$.

8. (a) Solve the three dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ (use variable-separable method).
   (b) Construct general solution of two dimensional Laplace's equation in plane polar co-ordinates.

9. A rod of length l with insulated sides, is initially at a uniform temperature u_0. Its ends are suddenly coated to 0°c, are kept at that temperature. Show that the temperature function $u(x, t)$ is given by $u(x, t) = \sum_{n=1}^{\infty} b_n Sin \frac{n\pi x}{l}\, e^{-\frac{c^2 n^2 \pi^2}{l^2} t}$.

10. Solve the following boundary value problem : $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 \le x \le l, t \ge 0$. Subject to the boundary conditions $\left.\begin{array}{l} u\,(0, t) = 0, t > 0 \\ \frac{\partial u}{\partial x}(l, t) = 0, t > 0 \end{array}\right\}$ and initial conditions $u(x, 0) = \begin{cases} x, & 0 \le x < \frac{1}{4} \\ \frac{l}{2} - x, & \frac{l}{4} \le x < \frac{l}{2} \\ 0, & \frac{l}{2} \le x < l \end{cases}$ and $\frac{\partial u}{\partial t}(x, 0) = 0, 0 \le x \le l$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XII

(Analytical Dynamics)

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Explain the terms (i) degrees of freedom (ii) constraints and (iii) generalized co-ordinates. Classify the dynamical systems based on different types of constraints.
   (b) Explain the difference between possible displacement and virtual displacement. Give examples of each of these kind of displacements.

2. (a) Derive Lagrange's equations of motion from Hamiltons' canonical equations.
   (b) Apply Lagrange's equations of motion to find the equations of motion of a simple pendulam.

3. (a) Formulate generalized components of momentum in terms of derivatives of cartesian co-ordinates and kinetic energy with respect to generalized co-ordinates.
   (b) Obtain the equation of motion of a system of two particles of unequal masses connected by an inextensible string passing over a small smooth pulley.

4. Define Hamilton's function and find differential equations for Hamilton's function.

5. Discuss the motion of spherical pendulam deduced from Hamilton's canonical equation of motion.

6. (a) Explain principal co-ordinates and normal mode of vibration.
   (b) A uniform rod AB of length 8a is suspended from a fixed point C by means of a light inextensible string of length 13a attached to B. If the system is slightly disturbed in a vertical plane. Show that $\theta + 3\phi \; and \; 12\theta - 13\phi$ are principal co-ordinates where θ and ϕ are the angles which the rod and string respectively make with the vertical.

7. (a) Explain canonical transformation and discuss the conditions for a transfromation to be canonical.
   (b) Show that the transformation $Q = q \tan p$, $P = \log \sin p$ is canonical. Hence, find the generating function $F(q, Q)$.

8. (a) Define Lagrange bracket. Show that the Lagrange bracket is invariant under canonical transformation.
   (b) Prove that the Lagrange's bracket does not obey the commutative law of algebra.

9. (a) Derive Hamilton's characteristic function from the Hamilton-Jacobi equation and give physical significance of this function.
   (b) Show that Hamilton Principle function $S = \int L dt$, is a solution of Hamilton-Jacobi equation.

10. (a) Derive Euler's equations of a motion for the motion of a rigid body about a fixed point.
    (b) A uniform sphere rolls without slipping on the rough interior of a fixed vertical cylinder of greater radius. Discuss the motion of the sphere.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–XIII
(Fluid Mechanics)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Find the equation of continuity in the cylindrical polar co-ordinates.

   (b) Show that the variable ellipsoid $\frac{x^2}{a^2k^2t^4} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$ is a possible form for the boundary surface of a liquid at any time t.

2. (a) Define stream line and path line. Construct the differential equation of a stream line.

   (b) The velocity $\vec{q}$ in a three dimensional flow field of an incompressible fluid is given by $\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}$. Determine the equation of stream line passing through the point (1, 1, 1).

3. (a) Derive Euler's equation of motion in spherical polar co-ordinates.

   (b) Let the air obeying Boyle's law is in motion in an uniform tube of small section; prove that if ρ be the density and v the velocity at a distance x from a fixed point at time t, then $\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\{(v^2 + k)\rho\}$.

4. (a) If the forces are conservative, motion is steady and ρ is a function of pressure p only, the equation of motion is $\int \frac{dp}{\rho} + \frac{1}{2}q^2 + \Omega = c$, where c is absolute constant and force $\vec{F} = -\nabla\Omega$.

   (b) A quantity of liquid occupies a length *2l* of a straight tube of uniform bore under the action of a force to a point varying as the distance from that point. Determine the motion and pressure.

5. (a) If w is an analytic function of z (complex variable), then its real part is the velocity potential and imaginary part is the stream function of an irrotational two dimensional motion. Prove this statement.

   (b) In two dimensional motion, show that if these stream lines are on confocal ellipses $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$, then $\psi = A \log\left(\sqrt{a^2 + \lambda} + \sqrt{b^2 + \lambda}\right) + B$ and the velocity at any point is inverrely proportional to the square root of aree of rectangle under the focal radii of the point, where ψ is a stream function.

6. (a) Explain the following terms :—
   (i) Two-dimensional image-system, (ii) Source, (iii) Sink and (iv) Doublet.

   (b) Find the stream functions $\psi(x, y, t)$ for the given velocity field $V = Ut$, $v = x$.

7. (a) Find the kinetic energy of an infinite mass of liquid moving irrotationally at rest at infinity and bounded internally by a solid surface S and externally by a large surface S'.

   (b) The space between two fixed co-axial circular cylinders of radii 'a' and 'b' and between two planes perpendicular to the axis and distant 'c' apart, is occupied by liquid of density ρ. Show that the velocity potential of a motion whose K.E. shall equal to a given quantity T, is given by A, where A is given by $\pi \rho A^2 c \log\left(\frac{b}{a}\right) = T$.

8. (a) Describe the motion of a sphere in an uniform stream.

   (b) An infinite homogeneous liquid is flowing steadily past a rigid boundary consisting partly of the horizontal plane $y = 0$ and partly of a hemispherical boss $x^2 + y^2 + z^2 = a^2$, with irrotational motion which tends, at a great distance c from the origin to uniform velocity U parallel to the z-axis. Find the velocity potential and the surface of equal pressure.

9. (a) Derive the rate of strain tensor or deformation tensor in the most general motion of fluid.

   (b) Prove that the vector Ω of an incompressible viscous fluid moving under no external force satisfied the differential equation $\frac{D\Omega}{Dt} = (\Omega \cdot \nabla)\vec{q} + u\nabla^2\Omega$, where μ is the coefficient of viscosity.

10. (a) Describe the generalized plane Couette flow.

    (b) Show that in the two dimensional motion of a viscous liquid, acted on by a conservative system of forces, the stream function satisfies the equation $\left(\nu\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2\psi = \frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)}$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–XIV
(Operations Research)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a convex set in the space R^n. Show that the set $S = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 1, x_1 + x_2 \geq 1\}$ is a convex set in R^2.

   (b) It is given that there are n different foods $F_1, F_2, --- F_n$ and m different nutrients contained in the food stuffs. Further it is given that (i) Cost of one unit of food stuff $j = c_j \ (j = 1, 2, ---, n)$, (ii) no. of nutrient i in one unit of food stuff $j = a_{ij} \ (i = 1, 2, ---, m;$ $j = 1, 2, ---, n)$ and (iii) minimum daily requirement of nutrient $i = v_i \ (i = 1, 2, 3, ——, m)$. Formulate the L.P.P. of minimization kind of daily requirement of the nutrients.

2. (a) Introduce the concept of basic feasible and non-basic feasible solution.

   (b) Solve the following L.P.P. by graphical method (if solvable);
   Maximize $z = 0.75\ x_1 + x_2$
   Subject to $x_1 - x_2 \geq 0 \ \ and \ \ -5x_1 + x_2 \leq 1, \ \ where \ \ x_1 \geq 0, \ x_2 \geq 0$

3. Use simplex method to solve the L.P.P.
   Maximize $z = 8x_1 + 19x_2 + 7x_3$
   Subject to $3x_1 + 4x_2 + x_3 \leq 25, \ \ x_1 + 3x_2 + 3x_3 \leq 50$. Comment on the result.

4. Apply the two-phase method to solve the L.P.P.
   Minimize $z = x_1 - 2x_2 - 3x_3$
   Subject to $-2x_1 + x_2 + 2x_3 = 2, \ \ 2x_1 + 3x_1 + 4x_3 = 1 \ \ (x_i \geq 0, i = 1, 2, 3)$

5. (a) State and prove Basic Duality theorem.

   (b) Write down the dual of the following L.P.P. and solve : Minimize $z = -2x_1 + 9x_2 + x_3$; subject to constraints $x_1 + 4x_2 + 2x_3 \geq 5, \ \ 3x_1 + x_2 + 2x_3 = 4 \ \ and \ \ x_1, x_2, x_3 \geq 0$.

6. Explain dual simplex method for the solution of a L.P.P. and apply it to solve the L.P.P. given as under : Maximize $z = -2x_1 - x_3$
   Subject to $x_1 + x_2 - x_3 \geq 5, \ \ x_1 - 2x_2 + 4x_3 \geq 8 \ \ and \ \ x_1 \geq 0, \ \ x_2 \geq 0, \ \ x_3 \geq 0$.

7. Give the detail account of sensitivity of the optimal solution of a L.P.P. with regard to change in the coefficients c_j of the objective function.

8. (a) Explain the following terms with regard to game problem. (i) Two persons zero sum of game, (ii) Strategies, (iii) Pay off matrix, (iv) Value of the game.

   (b) Test the solvability of the game having pay off matrix $\begin{bmatrix} 1 & 6 \\ 4 & 5 \\ 5 & 3 \end{bmatrix}$.

9. Explain Fibonacci method of solution for having non-linear objective function of non-linear programming problem.

10. Use Lagrange's multiplier method to obtain the solution of non-linear programming problem :
    Maximize $z = x_1^2 + 3x_2^2 + 5x_3^2$
    Subject to $x_1 + x_2 + 3x_3 = z, \ \ 5x_1 + 2x_2 + x_3 = 5 \ \ where \ \ x_1, x_2, x_3 \geq 0$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–XV
(Tensor Algebra, Integral Transform, Linear Integral Equations, Operation Research Modeling)
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a relative tensor of weight w. Prove that the equations of transformation of a relative tensor possess the group property.

   (b) If the tensors (a_{ij}) and (b_{ij}) are symmetric and u^i, v^i are components of contravariant vectors satisfying the equations $\left.\begin{matrix}(a_{ij} - k\,b_{ij})\,u^i = 0\\(a_{ij} - k'\,b_{ij})\,v^i = 0\end{matrix}\right\}$ $i, j = 1, 2, 3, ----, n$ *and* $k \neq k'$.

   Show that $a_{ij}\,u^i\,v^j = 0$ *and* $b_{ij}\,u^i\,v^j = 0$.

2. (a) Introduce covariant derivative of a contravariant vector and formulate covariant derivative of a mixed tensor of rank two.

   (b) Establish the relations :–(i) $a^{ij}_{\ ,j} = \dfrac{1}{\sqrt{g}}\dfrac{\partial}{\partial x^j}\left(a^{ij}\sqrt{g}\right) + a^{jk}\begin{Bmatrix} i \\ jk \end{Bmatrix}$, (ii) $a^j_{i,\,j} = \dfrac{1}{\sqrt{g}}\dfrac{\partial}{\partial x^j}\left(a^j_k\sqrt{g}\right) + a^j_k\begin{Bmatrix} k \\ ij \end{Bmatrix}$.

3. (a) Define Laplace Transform and prove its linear property.

   (b) If *F(t)* is a function of class *A* and *L{F(t)} = f(s)*. Then prove that $L\{t^n F(t)\} = (-1)^n \dfrac{d^n}{ds^n} f(s)$, *n = 1, 2, 3, - - - -* .

4. (a) State convolution theorem and apply it to evaluate (i) $L^{-1}\left\{\dfrac{1}{(s+a)(s+b)}\right\}$, (ii) $L^{-1}\left\{\dfrac{s}{(s^2+a^2)^2}\right\}$.

   (b) If *f(s)* be the Fourier transform of *F(x)*, then show that $\frac{1}{a} f\left(s/a\right)$ is the Fourier transform of *F(ax)*.

5. (a) Describe t he construction of solution of the Fredholm integral equation of first kind.

   (b) Form an integral equation corresponding to the differential equation given by $\dfrac{d^2y}{dx^2} - Sinx\dfrac{dy}{dx} + ye^x = x$ under the initial conditions $y(0) = 1$ and $y'(0) = -1$.

6. (a) What is Volterra integral equation of first kind ? Find the resolvent kernel of the Volterra integral equation $f(x) = \int_0^\infty e^{x-t}\ u(t)dt,\ f(0) = 0$.

   (b) Explain the method of solution of Volterra Integral equation.

7. (a) Explain the technique of replacement of items that deteriote with time in 'Replacement Models'.

   (b) The following table gives the running costs per year and resale prices of a certain equipment whose purchase price is Rs. 5000/–.

Years	1	2	3	4	5	6	7	8
Running Cost (in Rs.)	1500	1600	1800	2100	2500	2900	3400	4000
Resale Value (in Rs.)	3500	2500	1700	1200	800	500	500	500

   At what year is the replacement due ?

8. Discuss deterministic model with instantaneous production (shortages allowed).

9. Make analysis of Poisson Queueing Systems Model $3(M/M/S):(\propto/F/F0)$ system.

10. If for a period of 2 hours in a day (8 AM–10 AM) trains arrive at the yard every 20 minutes, but the service time continuous to remain 36 minutes. Then calculate for the period.

    (a) the probability that the yard is empty.

    (b) average queue length, on the assumption that the line capacity for the yard is limited to 4 trains only.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XVI

(Programming in 'C')

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Explain different Data type used in C with an example for each.
   (b) What is the difference between a variable and a constant?
2. Explain Relational Operator, Conditional Operator and Evaluation Operator with examples.
3. Describe two different ways to utilize the increment and decrement operators? How do the two methods differ?
4. In what way does an array differ from an ordinary variable? How are individual array elements identified ?
5. Explain types of looping statements with examples. What is difference between while loop and do-while loop ?
6. Explain the concept of Global and local variables. Explain the difference between call y reference and call by value giving suitable examples.
7. Write a programme in C to find the roots of a quadratic equation.
8. Describe different Format specifiers and Escape sequence along with their usage and examples.
9. What is function? Are functions required when writing a C program?
10. Write short notes on any two :
    (a) Recursion
    (b) Break Statement
    (c) File Management in ‘C’
    (d) Pointers in ‘C’

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M.Sc. Mathematics, Part–II, Paper–XVI (Practical)

Counselling & Examination Programme, 2013

Practical Counselling Programme

Date	*Time*	*Venue*
30.08.2013 to 04.09.2013	12:00 Noon to 03:00 PM	Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001

Practical Examination Programme

Date	*Time*	*Venue*
05.09.2013	12:00 Noon to 03:00 PM	Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001