Nalanda Open University

Annual Examination - 2013

B.Sc. Mathematics (Honours), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least on from each group.

Group 'A'

1. (a) Define an equivalence relation. Show that a relation R on the set $N \times N$ is an equidence relation when (a, b) R (c, d) iff a + d = b + c.

   (b) What do you mean by a partially ordered set? Let G be a group and M be the set of all sub–groups of G. Define $A \leq B$ to mean $A \subseteq B$. Prove that M is a partially ordered set.

2. (a) Let $f : X \to Y$ be such that $f(A^c) = \{f(A)\}^C$ for all $A \subseteq X$, show that f is one–one.

   (b) Prove that any denumerable union of denumerable sets is denumerable.

Group 'B'

3. (a) Introduce Idempotent and Nilpdent matrix and give their examples with support.

   (b) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, find the value of $A^2 - 4A + 3I$, where I is the unit matrix with proper dimension.

4. (a) By giving the idea of Hermitian and skew–Hermitial matrices, prove that every square matrix is uniquely expressed as the sum of a Hermitian and skew-Hermitian matrix.

   (b) If A be a n-rowed square matrix, then show that (AdjA) A = A (AdjA) = $|A| I_n$, Where $|A|$ is the determinant of A and I_n is the unit matrix of order n.

Group 'C'

5. Define H.C.F of two integers which is denoted by (a, b). Prove that for a, $b \in Z$ there exist integers s and t such that (a, b) = as + bt.

6. (a) Define a binary operation on a non–empty set A. Construct two examples (with proof) that non–empty sets are not groups.

   (b) Prove that the set of residue classes of integers modullo a prime number form additive abelian and multicative abelian groups.

7. (a) In a group, every element possesses unique inverse. Prove this statement.

   (b) Show that the inverse of the product of two elements of a group in the product of their inverses in the reverse order.

Group 'D'

8. (a) Describe the relations between roots and Coefficients of an nth degree equation.

   (b) Find the condition that the roots $\alpha, \beta, \gamma, \delta$ of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ should be connected by the relation $\alpha\beta = \gamma\delta$.

9. Solve the cubic equation $x^3 + 6x^2 + 9x + 4 = 0$ by Cardon's method.

Group 'E'

10. (a) State and prove De–Moivre's theorem for integers.

    (b) Prove that $(i)^i = e^{-(4n+1)\pi/2}$

11. (a) State and establish the theorem of Gregories series.

    (b) If tan (x + iy) = $\cos\alpha + i\sin\alpha$, prove that $y = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$

NALANDA OPEN UNIVERSITY

B.Sc. Mathematics (Hons.)

PART–I, PAPER–II

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer Five Questions in all, selecting at least One Question from each Group.
All questions carry equal marks.

Group 'A'

1. (a) State and prove Rolle's Theorem.

   (b) If $Cos^{-1}\frac{y}{b} = \log\left(\frac{x}{a}\right)^n$, then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$.

2. (a) If $u = \tan^{-1}\frac{x^3+y^3}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

   (b) Evaluate $\lim\limits_{x \to 0} \frac{x e^x - \log(1+x)}{x^2}$.

3. (a) Find the radius of curvature at any point (r, θ) for the curve $r^n = a^n \; Cos n\theta$.

   (b) Determine the equation of asymptotes for the curve $x^3 - 2y^3 + xy\,(2x-y) + y(x-y) + 1 = 0$.

Group 'B'

4. Evaluate any two of the following integrals :—

   (a) $\int \frac{x^2 dx}{(x-1)^3 (x+1)}$ (b) $\int \frac{x dx}{(x+2)^{1/3}}$ (c) $\int \frac{dx}{(1+x)\sqrt{1+3x+x^2}}$

5. (a) Derive the reduction formula for the integral $\int Sin^n x \; dx$.

   (b) Evaluate the definite integral $\int_0^{\pi/2} \log \; Sin\, x \; dx$.

6. Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx \; dy}{\sqrt{1+x^2+y^2}}$.

7. Find the surface area and volume of the solid of revolution of the curve $y^2(a-x) = a^2 x$ about its asymptote.

Group 'C'

8. (a) Prove that from an external point, there are three normals drawn to the parabola $y^2 = 4ax$.

   (b) Trace the parabola $9x^2 - 24xy + 16y^2 - 50x - 100y + 225 = 0$.

9. (a) Find the equation of the normal at any point $P(\alpha)$ of a conic $\frac{l}{r} = 1 + e\; Cos\; \theta$.

   (b) An ellipse of semi-axes a and b touches the axis of x at the origin. Prove that the locus of the centre is $x^2y^2 + (y^2 - a^2)(y^2 - b^2) = 0$.

Group 'D'

10. (a) Derive the equation of tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

    (b) Find the condition that the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ has three mutually perpendicular tangent planes.

11. (a) Find the equation of the right circular cylinder whose axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius a.

    (b) Derive the equation of the normal to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point (x_1, y_1, z_1).

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NALANDA OPEN UNIVERSITY

B.Sc. (Hons.), *PART–I*

Mathematics (Subsidiary), Paper–I

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer Five Questions in all, selecting at least One Question from each Group.
All questions carry equal marks.

Group 'A'

1. (a) State and prove Distributive Laws of set theory.
   (b) Let $f : A \to B$ and $g : B \to C$ be two bijective maps. Then, prove that $gof : A \to C$ is bijective and $(gof)^{-1} = f^{-1} o\, g^{-1}$.

2. (a) Prove that in a group, every element possesses unique inverse.
   (b) Define a cyclic group and show that every cyclic group is abelian.

3. (a) Show that the nth roots of unity form a geometric progression.
   (b) Evaluate $\underset{x \to 0}{Lim} \dfrac{3\,Sinx - Sin3x}{x - Sinx}$.

4. (a) State and prove the theorem on Gregories series.
   (b) Prove that $\tan\left(i \log \dfrac{a - i\,b}{a + i\,b}\right) = \dfrac{2ab}{a^2 - b^2}$, where a and b are real quantities.

Group 'B'

5. (a) Date and prove Cauchy's Root Test.
   (b) Let $x_1 = 1,\ x_2 = \sqrt{2 + x_1},\ x_3 = \sqrt{2 + x_2},\ -----,\ x_{n+1} = \sqrt{2 + x_n},\ -----$. Show that the sequence (x_n) is Convergent having limit of Convergence 2.

6. (a) Find the condition that the circles $x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$ and $x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$, intersect orthogonally.
   (b) Show that the equation $2x^2 + 3y^2 - 4x + 5y + 4 = 0$ represents an ellipse. Hence, obtain its focii and eccentricity.

7. (a) Find the condition that the line $lx + my + n = 0$, is tangential to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.
   (b) Find the equation of pair of tangent lines drawn from $(1, 1)$ to $2x^2 + y^2 - 4x + 2y + 2 = 0$.

Group 'C'

8. (a) If $y = e^{a\,Sin^{-1}x}$, show that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = a^2 y$.
   (b) State and prove Maclaurin's series theorem.

9. (a) Evaluate $\underset{x \to 0}{Lim} \dfrac{xe^x - \log(1 + x)}{x^2}$.
   (b) Find the radius of curvature of the curve $x^2 y = a\,(x^2 + y^2)$ at the point $(-2a,\ 2a)$.

10. (a) Prove that $\vec{r} = (\vec{r} \cdot \hat{i})\,\hat{i} + (\vec{r} \cdot \hat{j})\,\hat{j} + (\vec{r} \cdot \hat{k})\,\hat{k}$.
    (b) If $\vec{r} = \vec{a}\ Cos\ wt + \vec{b}\ \ Sin\ wt$, then show that $\vec{r} \times \dfrac{d\vec{r}}{dt} = w\,(\vec{a} \times \vec{b})$.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics (Hons.)

PART–II, PAPER–III

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions, selecting at least one from each Group. All questions carry equal marks.

Group 'A'

1. Use Dedekind cut or otherwise prove that $\sqrt{2} \times \sqrt{3} = \sqrt{6}$.

2. (a) State and prove Canter–Dedekind theorem on order completeness theorem for real number system.
   (b) Apply Dedekind's theorem to deduce the theorem of least upper bound.

3. (a) Show that if x and y be any two positive real numbers, then there exists a positive integer n such that $nx > y$.
   (b) Define an open set in R(set of real numbers) and give two examples with proof.

4. (a) State and prove Bolzano-Weierstrass theorem.
   (b) If $I_1 = [a_1, b_1], I_2 = [a_2, b_2], ----$ be a sequence of closed bounded intervals in R such that $I_n \supseteq I_{n+1}$ for each $n \in N$ and $\underset{n \to \infty}{Lim}(b_n - a_n) = 0$. Then show that $\bigcap_{n=1}^{\infty} I_n$ contains precisely one point.

Group 'B'

5. (a) Define continuity of a function on an open interval. If a function f is continuous on a closed and bounded interval [a, b], then prove that it attains its bounds on [a, b].
   (b) Discuss continuity of $f(x) = e^{\frac{1}{x}} (x \neq 0)$ and $f(0) = 1$ at the origin.

6. (a) Define Beta function and evaluate a Beta function.
   (b) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

7. (a) Prove that every convergent sequence is bounded.
   (b) Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{nx}{n+1}\right)^n, \ x > 0$.

8. (a) State and explain Cauchy's Condensation Test.
   (b) Test the convergence of the series $(x > 0)$ $1 + \frac{2^2}{3^2}x + \frac{2^2.4^2}{3^2 5^2}x^2 + \frac{2^2.4^2.6^2}{3^2.5^2.7^2}x^3 + ---$ to ∞.

Group 'C'

9. (a) Introduce the concept of a vector space and give a suitable example with explanation.
   (b) If V be a vector space over the field F and $\alpha, \beta \in V$, then prove that $\beta + (\alpha - \beta) = \alpha$.

10. (a) Define a sub-space of a vector space.
    (b) Prove that the set W of the elements of the vector space $V_3(F)$ of the form $(x + 2y, y, 3y - x)$, where $x, y \in F$, is a sub=space of $V_3(F)$.

11. (a) Define Eigen values and Eigen vectors of a matrix. Prove that the Eigen vectors corresponding to distinct Eigen values of a matrix are linearly independent.

(b) Find the Eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics (Hons.)

PART–II, PAPER–IV

Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer Five Questions in all, selecting at least One Question from each Group.
All questions carry equal marks.

Group 'A'

1. (a) Solve any two of the following :—
   (i) $y = (1 + p)x + ap^2$ (ii) $y = px + p - p^2$ (iii) $xyp^2 - (x^2 - y^2)p - xy = 0$
   (b) Show that the system of parabola $y^2 = a(x + a)$ is self-orthogonal.
2. Solve the differential equations given below :—
   (i) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = x^3$ (ii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}$
3. (a) Use the variation of parameters to solve $\frac{d^2y}{dx^2} + a^2 y = Sec\, ax$.
   (b) Solve $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 3)y = e^{x^2}$.

Group 'B'

4. (a) Show that, $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}\right] = \left[\vec{a}, \vec{b}, \vec{c}\right]^2$.
   (b) If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors, $\vec{b}$ and $\vec{c}$ being non-parallel and such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}}{2}$. Find the angle which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$.
5. (a) Give the physical meaning of divergence and curl of a vector valued function.
   (b) Prove the following results :—(i) $\nabla(\vec{r}^2) = 2\vec{r}$, (ii) $\nabla\left\{div\left(\frac{\vec{r}}{r}\right)\right\} = -\frac{2}{r^3}\vec{r}$.
6. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where S is the surface of the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, $z = 0$, $z = 1$ and $\vec{F} = 2x^2\hat{i} - 4yz\,\hat{j} + zx\hat{k}$.

Group 'C'

7. (a) If all forces in a coplanar system are rotated about their points of application through the same angle in their own plane, then prove that their resultant passes through a fixed point in the body.
   (b) Three forces P, Q, R act along the sides of a triangle formed by the lines x + y = 1, y – x = 1 and y = 2. Find the equation of the line of action of the resultant force.
8. (a) State and prove converse of the principle of virtual work.
   (b) The middle points of opposite sides of a jointed quadrilateral are connected by light rods of lengths *l* and *l'*. If *T* and *T'* be tensions in there rods, then prove that $\frac{T}{l} + \frac{T'}{l'} = 0$.

Group 'D'

9. A particle moves in a straight line OA with an acceleration which varies as the distance from a fixed point o in the straight line and is always away from the fixed point. If the particle was initially at a distance 'a' from O and projected with the velocity V towards O. Discuss the motion.
10. (a) Formulate the tangential and normal velocities of a moving particle in plane.
    (b) A particle describe a catenary under a force which acts parallel to its arias. Find the law of the force and the velocity at any point of the path.
11. (a) Deduce the differential equation of motion of a particle moving in a central orbit in the form $p = \frac{h^2}{p^3}\frac{dp}{dr}$.

(b) Discuss the motion of a particle moving under inverse square law and show that the path is a comic section.

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NALANDA OPEN UNIVERSITY
B.Sc. (Hons.), *PART–II*
Mathematics (Subsidiary), PAPER–II
Annual Examination, 2013

Time : 3 Hours. **Full Marks : 80**

Answer Eight Questions in all, selecting at least One Question from each Group. All questions carry equal marks.

Group 'A'

1. Evaluate any two of the following :—

   (i) $\int \frac{dx}{a^2 - x^2} \, (x < a)$ (ii) $\int \frac{1 + x^{\frac{1}{2}}}{1 + x^{\frac{1}{3}}} \, dx$ (iii) $\int \frac{dx}{(1 + x)\sqrt{1 + 3x + x^2}}$

2. (a) Evaluate $\int_0^1 Sinx \, dx$ as the limit of sum. (b) Evaluate $\int_0^{\pi/2} \log Sinx \, dx$.

3. (a) Derive the reduction formula for $\int Sec^n x \, dx$.

   (b) Evaluate $\lim_{n \to \infty} \left[\frac{n^2}{(n^2 + 1^2)^{3/2}} + \frac{n^2}{(n^2 + 1^2)^{3/2}} + - - - - - + \frac{n^2}{\{n^2 + (n-1)^2\}^{3/2}} \right]$.

4. (a) Find the entire length of the cardoid $r = a(1 + Cos\,\theta)$.

   (b) Find the area between the curve $y^2(2a - x) = x^3$ and its asymptote.

5. Solve any two the differential equations as under :—

   (i) $p^2 - p\left(e^x + e^{-x}\right) + 1 = 0$ (ii) $y = (1 + p)x + ap^2$ (iii) $y = px + \log p$

6. What do you mean by orthogonal trajectory of a system ? Find the orthogonal trajectory of the family $r^n + a^n \, Cos\, n\theta$.

7. Determine the general solution of the following equations :—

   (i) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$ (ii) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = Cos3x$ (iii) $\frac{d^2y}{dx^2} + y = x^2 Cosx$

Group 'B'

8. (a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4x + 2z = 3$ and hence find the point of contact.

   (b) Find the equation of the right circular cone whose axis is the n–axis, vertex the origin and semi-vertical angle $\frac{\pi}{3}$.

9. Determine the equation of the right circular cylinder whose axis is $\frac{x}{1} = \frac{y}{0} = \frac{z}{2}$ and radius $\sqrt{7}$.

Group 'C'

10. (a) By giving the idea of a convex set, prove that the set $S = \{(x_1, x_2)/3x_1^2 + 2x_2^2 \leq 6\}$ is a convex set.

    (b) What is the idea of convex combination of vectors ? Show that any point that can be expressed as a convex combination of two points in R^n, lies on the lien segment joining the two points.

11. Write short notes on :—

    (a) Neghbourhood of a point. (b) Interior point. (c) Boundary point.
    (d) Extreme point. (e) Open set.

Group 'D'

12. Use vector method to find the resultant of a system of coplanar forces and then derive the conditions of equilibrium.

13. Three forces each equal to P, act along the sides of the triangle ABC taken in order. Find the resultant force and equation of line of action of the resultant and hence deduce its distance from A and its point of intersection with BC.

Group 'E'

14. (a) State and prove the principle of conservation of linear momentum.

    (b) Compound two simple harmonic motions along the same straight lines.

15. (a) A particle moves in a straight line from rest and an acceleration which is proportional to the distance from a fixed point O in the straight line and is always away from O. Discuss the motion.

    (b) The energy of a stretched elastic string is equal to half the product of the tension and the extension.

16. (a) Find the velocities and accelerations in the intrinsic co-ordinates.

    (b) A particle is projected with a velocity $2\sqrt{ga}$ so that it just clears two walls of equal height a which are at a distance 2a from each other. Show that the time of passing between the walls is $2\sqrt{a/g}$.

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Nalanda Open University
Annual Examination - 2013
B.Sc. Mathematics (Honours), Part-III
Paper-V

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) Define open set. Prove that a sub-set G of a metric space x is open iff G is an union of open spheres.
   (b) Prove that the union of any family of open sets in (x, d) a metric space, is open.
2. (a) Let (x, d) be a metric space. Prove that the intersection of any family of closed in X, is closed.
   (b) In a metric space, prove that each closed sphere is a closed set.
3. (a) Let x and y be metric spaces and f a map of x into y. Show that f is continuous iff $f(\overline{A}) \subseteq f(\overline{A})$ for every sub-set A of X.
   (b) Let X and Y be metric space and f is a mapping of X into Y. if f be a constant mapping, then show that f is continuous.
4. (a) In a topological space, give a characterization of continuity in terms of open sets.
   (b) Show that every sub-space of a Hausdorff space is Hausdoff.

Group-B

5. (a) Prove that every bounded monotonic increasing function defined on a closed interval is Riemann integrable.
   (b) Let f(x) be defined on [0, 1] by the condition for n to be positive integer, then f(x) $=(-1)^{n-1}$ where $\frac{1}{n+1} \le x \le \frac{1}{n}$ is R integrable and $\int_0^1 f(x)dx = \log 4 - 1$.
6. (a) If f and g are two bounded and R-integrable functions on [a, b], then show that f+g is R-integrable on [a,b] and $\int_a^b \{f(x)+g(x)\}dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.
   (b) Prove that $\lim_{n\to\alpha}\left\{\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\text{.........}+\frac{1}{2n}\right\}=\frac{\pi}{4}$

Group-C

7. Discuss the Convergence of $1+\frac{1}{3^\alpha}-\frac{1}{2^\alpha}+\frac{1}{5^\alpha}+\frac{1}{7^\alpha}-\frac{1}{4^\alpha}+\frac{1}{9^\alpha}+\frac{1}{11^\alpha}-\frac{1}{6^\alpha}+\text{.........to}\,\alpha$, for different values of α.
8. (a) State and prove Pringshiem's Theorem on double series.
   (b) If a double series is absolutely convergent, then show that it is also convergent,

Group-D

9. (a) Let N be a non-zero normed linear space. Show that N is Banach space iff f{x:||x||=1} is complete.
   (b) Prove that a normal linear space is a Banach iff every absolutely summable series is summable.
10. (a) By taking R^2 to be real normed linear space given by $\|x\| = \sqrt{x_1^{\,2} + x_2^{\,2}}$, where $x=(x_1,x_2)$, show that T is a Continuous linear transformation $T: R^2 \to R$ defined by $T(x_1,x_2)=x_1$.
    (b) Give definition and example of an inner produce space with illustration.
11. (a) Define Hilbert space. Show that $l^n_{2\ \text{space}}$) with inner product of two vectors $x=(x_1, x_2, \text{.....}, x_n)$ and $y=(y_1,y_2, \text{.....}y_n)$ defined by $(x,y)=\sum_{i=1}^{n} x_i \overline{y}_i$, is a Hilbert space, where l^n_2 = $\{x=(x_1, x_2, \text{......}, x_n)/x_1, x_2, \text{.....}, x_2 \in C\}$.
    (b) Consider the linear space P[0,1] of all real valued polynomials on [0,1] with the inner product (f,g)= $\int_0^1 f(t)g(t)dt$ where $f,g \in P[0,1]$. Show that it is an inner product space, but not a Hilbert space.

Nalanda Open University
Annual Examination - 2013
B.Sc. Mathematics (Honours), Part-III
Paper-VI

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) Define an automorphism of a group G. Let $x \in G$, then prove that the function f defined by $f(g) = x^{-1} g x$ for g in G, is an automorphism of G.
   (b) Let C(G) denotes the centre of a group G and I(G) be the set of inner automorphisms on G. Then prove that $G/C(G) \cong I(G)$.
2. (a) Define a ring homomorphism. Let $f : R \to T$ be a homomorphism of a ring R onto a ring T. Then show that f is an isomorphism iff Kernel f={0}.
   (b) Show by an example that if I and J are ideals of a ring R, then I U J is not an ideal of R.
3. (a) Show that any ring can be embedded in a ring with unity.
   (b) Define Principal ideal ring and show that the ring of integers, is a principal ideal ring.
4. In the ring F[x], show that the principal ideal generated by the polynomial x-s, where $s \in F$, is both a prime ideal and maximal ideal.

Group-B

5. State and prove Cantor's Theorem.
6. (a) Define sum and product of cardinal numbers and gibe examples of each with supportive arguments.
   (b) Prove that $2^{\lambda_0} = C$ (symbols have their usal meanings).
7. (a) Define maximal and minimal elements of partial ordered set. Let A = {3,4,5,8,9} be ordered by x divides y. Find its maximal and minimal elements.
   (b) Introduce the concept of order types and construct the product of two order types.

Group-C

8. (a) Define a partition of a set and equivalence classes. Show that the equivalence classes are either disjoint or same.
   (b) Among any (n+1) integers not exceeding 2n, show that there must be an integer that divides one of the other inlegers.
9. (a) Explain the conditions of Bernoullis' trials.
   (b) Prove that ${}^{n}P_{r} = (n - r + 1)\,{}^{n}P_{r-1}$

Group-D

10. (a) Show that if $f(Z) = u(x, y) + i\,v(x, y)$ is differentiable at any point $Z = x + iy$, then the partial derivatirbs u_x, u_y, v_x, v_y exist and satisfy Canchy-Riemann differential equataions.
    (b) If $f(Z)$ is an analytic function of Z, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(Z)|^2 = 4\,|f^1(Z)|^2$.
11. (a) State and prove Canchy's integral formula.
    (b) Evaluate $\int \frac{e^{2Z} dZ}{(Z+1)^4}$, where C is the circle $|Z| = 3$.
12. (a) For the function $f(Z) = \frac{2Z^3 + 1}{Z^2 + Z}$, find Taylor's series valid in the neghbourhood of the point Z =i.
    (b) Explain singularities of a function. What kind of singularity has the function $Cos\left(\frac{1}{Z}\right)$ at $Z = 0$?

Nalanda Open University
Annual Examination - 2013
B.Sc. Mathematics (Honours), Part-III
Paper-VII

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) Define a feasible solution of a Linear Programming Problem. Prove that the set of all feasible solution of a L.P.P from a convex set.
   (b) Solve the following L.P.P. graphically:
   $\text{Max } Z = 4x_1 + 7x_2$
   Subject to: $x_1 + 2x_2 \le 20$, $x_1 + x_2 \le 15$, $x_2 \le 8$; where $x_1 \ge 0,\ x_2 \ge 0$

2. (a) Introduce degenerate and non-degenerate basic solution. Obtain all basic solution of the system: $x_1 + 2x_2 + x_3 = 4,\ 2x_1 + x_2 + 5x_3 = 6$ and specify degenerate and non-degenarte solutions.
   (b) Use simplex method to solve the L.P.P.
   Max $Z = 3x_1 - 2x_2$,
   Subject to the constraints $x_1 + x_2 \le 4,\ x_1 - x_2 \le 12$ and $x_1 \ge 0,\ x_2 \ge 0$.

3. From the initial basic feasible solution of the transportation problem by using matrix minima method:

	W_1	W_2	W_3	W_4	Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Require-ment	5	8	7	14	34

Group-B

4. Find a necessary and sufficient condition for the integrability of the total differential equation $Pdx + Qdy + Rdz = 0$

5. (a) Solve the differential equation $z(z-y)dx + z(z+x)dy + x(x+y)dz = 0$.
   (b) Apply charpit's method to find the complete integral of px + qy = pq.

6. (a) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y^2} - 15\frac{\partial^2 z}{\partial y^2} = 12xy$.
   (b) Construct the general solution of the Lagrange's linear equation $px(y^2 - z^2) - qy(z^2 - x^2) = z(x^2 - y^2)$, by forming its auxilliary equations.

7. (a) Use Monge's method to find the complete solution of the equation $2x^2 r - 5xys + 2y^2 t + 2(px + qy) = 0$
   (b) Find the orthogonal projection on the x-z plane of the curves which lie on the paraboloid $3z = x^2+y^2$ and satisfy the equation 2dz=(x+z)dx+ydy.

Group-C

8. (a) Find the attraction of a uniform sphere at an external point.
   (b) A frustum of a uniform thin hallow cone attracts a particle placed at the Vertex. Show that the attraction is $2\pi\rho \sin\alpha \operatorname{Cos}\alpha \log\frac{R}{r}$, where R and r are the radii of circular ends, α the semi verticle angle and ρ the surface density of the cone.

9. (a) State and establish Laplace equation in Cartesian Co-ordinates form.
   (b) Prove that the half of the potential of a uniform spherical shell at an external point 0, is due to that portion of the sphere which is nearer to 0.

Group-D

10. (a) Prove that the difference between pressures at two points of a homogeneous fluid varies as the depth of one point below the other.
    (b) A hemi-spherical vessel filled with water, is placed in an invested position on a horizontal table. Find the resultant thrust of the water on the vessel.

11. (a) Find the depth of the centre of pressure of a circular area of radius 'a' immersed vertically in a homogeneous liquid with its centre at a depth h below the free surface.
    (b) A rod of small cross section and density ρ has a small portion of a metal of weight $\frac{1}{n}$th that of the rod attached to one extremity. Prove that the rod will float at an angle (inclination) in a liquid of density σ if $(n+1)^2\rho = n^2\sigma$.

Nalanda Open University
Annual Examination - 2013
B.Sc. Mathematics (Honours), Part-III
Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *five* Questions. (Calculator not allowed)

1. (a) Evaluate (i) $\Delta^3(1-x)(1-2x)(1-3x)$ (ii) $\Delta^n e^{ax+b}$, where a and b are constants.
   (b) Derive Largrange's Interpolation formula and apply it to find $\log_{10}656$, where $\log_{10}654=2.8156$, $\log_{10}658=2.8182$, $\log_{10}659=2.8189$ and $\log_{10}661=2.8202$.

2. (a) Explain divided differences of a data and prove that the value of any divided difference is independent of the order of arguments.
   (b) Express $f(x)=2x^3-3x^2+3x-10$ in factorial notations, the interval of differencing being unity.

3. (a) Introduce Gauss Central difference forward interpolation formula.
   (b) Find the value of u_{28} by Striling formula, when $u_{20} = 49225$, $u_{25} = 48316$, $u_{30}=47236$, $u_{35}=45926$ and $u_{40}=44306$.

4. (a) Derive Simpson's $\frac{3}{8}$ rule for numerical integration.
   (b) By using Weddle's rule, evaluate $\int_0^{10}\frac{dx}{1+x}$, by dividing the range into eight equal parts.

5. Solve the following difference equations:
   (a) $u_{x+2}-4u_{x+1}+13u_x=0$ (b) $u_{x+2}-3u_{x+1}+2u_x=2^x$.

6. Describe Euler's method of solution for differential equation. Hence, find approximate value of y for x=0.1, given that $\frac{dy}{dx}=\frac{y-x}{y+x}$, when y=1 for x=0.

7. (a) Use Gauss-Jordan Method to solve the system of linear equations: $x_1+2x_2+x_3=8$, $2x_1+3x_2+4x_3=20$ and $4x_1+3x_2+3x_3=16$. (Take initial condition x=0, y=0, z=0)
   (b) Explain Gauss-Seidel Method for construction of solution of a system of linear equations and elaborate by means of suitable example.

8. (a) Use Group Relaxation Method to solve the following system: $-10x+2y+2z+4=0$, $x-10y+2z+10=0$ and $x+y-10z+45=0$.
   (Hint: Take initial condition x=0 , y= 0 and $z=0$)
   (b) Describe Convergent Gauss-Seidel Method.

9. (a) Apply analytic method for finding roots of an equation, based on Rolle's theorem and demonstrate on $3x-\sqrt{1+\sin x}=0$.
   (b) Solve $x^3-9x+1=0$ for the roots lying between x=2 and x=4 by bisection method.

10. (a) Discuss Newton-Raphson's Method to obtain approximate value of root of f(x)=0.
    (b) Use synthetic division to solve $f(x)=x^3-x^2-(1.001)x+0.9999=0$, in the neghbourhood of x=1.

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