Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Honours), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

Answer any ***Five*** *questions.* ***Question No. 1 is Compulsory.*** *All questions are of equal value*

Group - A

1. (a) Define Indexed family of sets : Establish generalized associative laws

   (i) $A \cup (\bigcup_{i \in I} B_i) = \bigcup_{i \in I} (A \cup B_i)$ and (ii) $A(\bigcup_{i \in I} B_i) = \bigcup_{i \in I} (A \cap B_i)$

   (b) Define relation on a set (non-empty) and introduce domain and range of a relation.

2. (a) If R_1 and R_2 are two relations on a non-empty set A. Prove that $R_1 \cap R_2$ is also a relation on A

   (b) State and prove Fundamental theorem on equivalence relation.

Group - B

3. (a) Introduce the concept of a Lattice and give an example of a partial ordered set which is not a Lattice.

   (b) Let $f: X \to Y$ and $A \subseteq X$, $B \subseteq X$, then show that $f(A \cap B) = f(A) \cap (B)$ does not always hold. Substantiate by means of an example.

4. What do you mean by Countable and uncountable sets? Prove that Countable union of Countable set is Countable. Show that set of real numbers is not Countable.

5. (a) By giving idea of transpose of a matrix, prove that the transpose of transpose of a matrix A is itself.

   (b) Prove that the matrix is a Hermitian.

$$A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & 0 \\ 2 & -1 & 0 \end{bmatrix}$$

Group - C

6. If H and K be two sub-groups of a group G. Then prove that HK is a sub-group of G iff HK=KH.

7. (a) What is a normal sub-group of group G. Prove that a sub-group N of group G is normal iff $gNg^{-1} = N$ for all $g \in G$.

   (b) State and prove fundamental theorem of homomorphism of groups.

8. Define integral domain and field. Prove that every finite integral domain is a field.

Group - D

9. State and prove the fundamental theorem of algebra.

10. (a) Describe Euler's method of solution of biquadratic equation.

    (b) Use Cardon's method to solve the equation $2x^3+3x^2+3x+1=0$

Group - E

11. (a) If n is a positive integer, prove that $(1+i)^n+(1-i)^n = 2^{\frac{n}{2}+1} \operatorname{Cos} \frac{n\pi}{4}$.

    (b) Find the expansion of Sin $\propto$ in terms of ascending powers of $\propto$.

12. (a) Prove that $\tan(i \log \frac{a-ib}{a+ib} = \frac{2ab}{a^2-b^2})$, where a and b are distinct real numbers.

(b) If $\sin^{-1}(x+iy) = \propto + i\beta$. Prove that (i) $\frac{x^2}{Cos\lambda^2\beta} + \frac{y^2}{Sin\lambda^2\beta} = 1$ (ii) $\frac{x^2}{Sin^2 \propto} - \frac{y^2}{Cos^2 \propto} = 1$

Examination Programme, 2014
B.Sc (Part – I) All Honours Subjects
Except Home Science and Geography Honours

Date	Papers.	Time	Examination Centre
12/6/2014	(Hons) P-I	8 to 11 am	Nalanda Open University, Patna
14/6/2014	(Hons) P-II	8 to 11 am	Nalanda Open University, Patna
16/6/2014	Rastrabhsha-100 or Hindi +Urdu 100	8 to 11 am	Nalanda Open University, Patna
18/6/2014	Botany (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
19/6/2014	Math (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
20/6/2014	Geography (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
21/6/2014	Chemistry (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
23/6/2014	Physics (Sub) P-I	8 to 11 am	Nalanda Open University, Patna
24/6/2014	Home Scince (Sub)-P I	8 to 11 am	Nalanda Open University, Patna
25/6/2014	Zoology (Sub) P-I	8 to 11 am	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Honours), Part-I

Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions. Selecting at least one from each group. All Question Carry equal marks.*

Group - A

1. State and prove Taylor's theorem in Cauchy's form of remainder.

2. (a) if y=$e^{a\sin^{-1}x}$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0.$

   (b) Derive the equation of the normal to the curve f(x,y)=0.

3. (a) if n be a homogeneous function of two variables x and y, then $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u.$

   (b) Evaluate the limit $x\to 0$ $\frac{\log x^2}{\log Cot^2 x}$.

Group - B

4. (a) Derive the radius of curvature formula in polar coordinates.

   (b) Find the asymptotes of the following curves.

   (i) $x^3+y^3=3axy$ (ii) $x^3-2x^2y+xy^2+x^2-xy+2=0.$

5. Evaluate any two of the following integrals :-

   (a) $\int\frac{dx}{x(x^2+1)^2}$ (b) $\int\frac{dx}{Cos\,x\,(2+3Sin\,x)}$ (c) $\int\frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$

6. (a) Derive the reduction formula for the integral $\int_0^{\frac{\pi}{2}}\sin^m x\,Cos^n x\,dx$.

   (b) Evaluate $\int_\alpha^\beta x\frac{dx}{\sqrt{(x-\alpha)\,(\beta-x}},\beta>\alpha$

7. Find the volume and the surface area generated by the revolution of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ about the x- axis.

Group - C

8. (a) Find the coonic represented by $9x^2-24xy+16y^2-124x+132y+324=0$.

   (b) Show that the line 4x+5y-14=0 touches the conic $x^2+4xy+y^2-2x+2y-15=0$ and hence find the point of contract.

9. Find the equation of chord of contact of tangents drawn from the point (r_1,θ_1) to the Conic $\frac{l}{r}=1+eCos\theta.$

Group - D

10. (a) Construct the equation of the sphere described on the line segment joining two given points A$(x_1,y_1\;z_1)$ and B $(.x_2,y_2,z_2)$ as diameter.

    (b) Find the equation of a sphere passing through four point ((0, 0, 0), (- a, b, c), (a,-b, c) and (a,b,-c).

11. (a) Define enveloping Cone to a surface. Hence, derive the equation of the enveloping Cone of the surface $x^2+y^2+z^2=a^2$, whose vertex is the point (x_1,y_1,z_1).

    (b) Derive the equation of the cylinder generated by lines parallel to a fixed line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$, the guiding curve being $ax^2+2hxy+by^2+2gx+2fy+c=0,z=0.$

Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Subsidiary), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions. Selecting at least one from each group. All Question Carry equal marks.*

Group - A

1. (a) Give the concept of a relation from a set A to a set B and also a relation on A.
   (b) Give example of at least one of these kinds of relations.
2. (a) Define an equivalence relation on a non-empty set X. If R is an equivalence relation on X, prove that R^{-1} (inverse relation of R) is also an equivalence relation on X.
   (b) A relation R is defined on the set N × N as follows : (a, b) R (c, d) iff $a + d = b + c$ Show that R is an equivalence relation on N × N.
3. Introduce one-one onto mapping. If $f : A \to B$ and $g : B \to C$ be one-one onto maps. Then show that the composite map $gof : A \to C$ is also one-one onto and $(gof)^{-1} = f^{-1}og^{-1}$.
4. (a) Define a group. Prove that every group has unique identity element.
   (b) What is a countable set? Prove that any denumerable union of denumerable sets, is denumerable.

Group - B

5. (a) State and prove De-Moivre's Theorem.
   (b) Expand $\frac{1 - Cosx}{Sinx}$ in ascending powers of x upto three places.
6. (a) Show that $\frac{\pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + ...$ to ∞.
   (b) Define six hyperbolic functions and prove that $\text{Cos}h^2x - \text{Sin}h^2x = 1$.

Group - C

7. (a) Prove that a monotonic increasing and bounded sequence tends to its least upper bound.
   (b) State and prove comparison test for the convergence of series.
8. (a) Find the equation of the circle passing through origin and cutting each of the circles $x^2 + y^2 - 6x + 8 = 0$ and $x^2 + y^2 - 2x - 7 = 0$ orthogonally.
   (b) Find the centre and eccentricity of the conic : $9x^2 - 4xy + 6y^2 - 14x - 8y + 1 = 0$

Group - D

9. (a) If $y = a \text{ Cos} (\log x) + b \text{ Sin} (\log x)$, then show that $x^2y_2 + xy_1 + y = 0$
   (b) State and prove Euler's theorem on homogeneous function of two variable.
10. Evaluate any **Two** limits :
    (a) $\lim_{x \to 0} \frac{\sqrt{1 + x - 1}}{\sqrt{x}}$ (b) $\lim_{x \to 0} x \sin x$ (c) $\lim_{x \to 0} \sin x \times \log x$
11. (a) Give the geometrical meaning of scalar triple product and explain the condition for its vanishing.
    (b) Find the gradient and the unit vector normal to the surface $x^2 + y^2 + z^2 = 1$ at the point (0, 0, 0).

Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Honours), Part-II

Paper-III

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions at least one from each group. All questions carry equal marks.*

Group - A

1. Describe Dedekind Cut in detail and also discuss types of Cut.
2. (a) Explain modified form of Dedekind's Cut.
   (b) Prove that $\sqrt{2}+\sqrt{3}$ is an irrational number.
3. (a) Establish Commutative law of addition of Cuts.
   (b) Prove that there is no rational number of such that $q^3 = 2$.

Group - B

4. (a) State and prove the theorem of greatest lower bound.
   (b) By introducing compete ordered field, prove that the set of rational number is not complete field.
5. (a) Define open set in the set of real numbers R. Prove that any non-empty open set is union of open intervals.
   (b) Test the set $\left\{\frac{1}{2^n} / n \in N\right\}$ for open and closed sets.
6. State and prove Heine-Barel Theorem.

Group - C

7. (a) Show that the function f defined by $f(x) = 1$, when x rational and $f(x) = -1$, when x is irrational, is totally discontinuous.
   (b) Prove that if f is continuous on $[a, b]$ and $f(a)$, $f(b)$ have opposite signs, then there exists a point $x \in [a, b]$ for which $f(x)$ vanishes.
8. (a) Prove that the limit of Convergent sequence is unique.
   (b) Test the Convergence of the series $\sum_{n=1}^{\alpha}\left(\frac{nx}{n+1}\right)^n, (x > 0)$.
9. (a) Define dependent and independent vectors in a vector space V(F). Produce an example of dependent vectors.
   (b) By giving an idea of a basis, prove that {(1, 2, 1), (2, 1,0), (1, –1, 2)} forms a basis of R^3.
10. (a) State and prove Cayley-Hamilton theorem.
    (b) Test the solvability of following system of non-homogeneous equations $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 6$ and find the solution of the system by matrix method.

Examination Programme, 2014

(Bachelor Of Science (Part-II)

All Subjects Except B.Sc Geography & Home Science (Honours)

Date	Paper	Time	Name of Examination Centre
21/5/2014	HONOURS PAPER – III	3.30 to 6.30 pm	Nalanda Open University, Patna
23/5/2014	HONOURS PAPER – IV	3.30 to 6.30 pm	Nalanda Open University, Patna
27/5/2014	(SUB.) (Mathematics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
28/5/2014	(SUB.) (Home Science- II)	**12.00 to 3.00 pm**	Nalanda Open University, Patna
29/5/2014	(SUB.) (Chemistry - II)	8.00 to 11.00 am	Nalanda Open University, Patna
30/5/2014	(SUB.) (Zoology - II)	8.00 to 11.00 am	Nalanda Open University, Patna
31/5/2014	Hindi 100 orUr 50+Hn50	**3.30 to 6.30 pm**	Nalanda Open University, Patna
02/6/2014	(SUB.) (Botany - II)	8.00 to 11.00 am	Nalanda Open University, Patna
02/6/2014	(SUB.) (Physics- II)	8.00 to 11.00 am	Nalanda Open University, Patna
04/6/2014	(SUB.) (Geography -II)	8.00 to 11.00 am	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Honours), Part-II

Paper-IV

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions at least one from each group. All questions carry equal marks.*

Group - A

1. (a) Solve any two of the following differential equations :

   (i) $y = 2px + 4xp^2$ (ii) $p^3 - p^2y - 1 = 0$ (iii) $y = x(p + p^2)$

   (b) Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$.

2. Describe the method of construction of general solution of linear differential equations having constant coefficients :

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + ...P_{n-1} \frac{dy}{dx} + P_n y = Q$$

Where $P_1, P_2, ... P_{n-1}, P_n$ are all constants and Q is a function of x.

3. (a) Solve $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ by reduction into normal form.

   (b) Solve the above differential equation by the method of variation of parameters.

Group - B

4. (a) By explaining scalar triple product, give the geometrical explanation of it.

   (b) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

5. Show that the plane through the points $P(\vec{r}_1), Q(\vec{r}_2), R(\vec{r}_3)$ has the equation :

   $[\vec{r}, \vec{r}_2, \vec{r}_3] + [\vec{r}, \vec{r}_3, \vec{r}_1] + [\vec{r}, \vec{r}_1, \vec{r}_2] = [\vec{r}_1, \vec{r}_2, \vec{r}_3]$

6. (a) If $\vec{a}$ is constant vector, prove that :

   (i) $\nabla(\vec{a}.\vec{r}) = \vec{a}$ (ii) $\nabla.(\vec{a} \times \vec{r}) = 0$

   (b) Find the moment about the point $\hat{i} + 2\hat{j} + 3\hat{k}$ of a force $\hat{i} + \hat{j} + \hat{k}$ acting through the point $-2\hat{i} + 3\hat{j} + \hat{k}$.

Group - C

7. Find the magnitude, direction and the equation of line of action of the resultant of a system of Co-planar forces.

8. (a) State and establish the principle of virtual work.

   (b) Two equal uniform rods AB and AC, each of length $2b$, are freely jointed at A and rest on a smooth vertical circle of radius a. If 2θ be the angle between them, then $b\sin^2\theta = a\cos\theta$.

Group - D

9. (a) State and prove the principle of energy.

   (b) A particle is projected under gravity with velocity $\sqrt{2ag}$ from a point at a height h above a level plane. Show that the angle of projection q for the maximum range is $2\sqrt{a(a+h)}$.

10. (a) Define the expressions for velocity and acceleration is polar Co-ordinates (By Vector method), in two dimensional space.

    (b) If the Central forces varies inversely as the cube of the distance from a fixed point. Find the orbit.

Examination Programme, 2014

(Bachelor Of Science (Part-II)

All Subjects Except B.Sc Geography & Home Science (Honours)

Date	Paper	Time	Name of Examination Centre
21/5/2014	HONOURS PAPER – III	3.30 to 6.30 pm	Nalanda Open University, Patna
23/5/2014	HONOURS PAPER – IV	3.30 to 6.30 pm	Nalanda Open University, Patna
27/5/2014	(SUB.) (Mathematics - II)	8.00 to 11.00 am	Nalanda Open University, Patna
28/5/2014	(SUB.) (Home Science- II)	**12.00 to 3.00 pm**	Nalanda Open University, Patna
29/5/2014	(SUB.) (Chemistry - II)	8.00 to 11.00 am	Nalanda Open University, Patna
30/5/2014	(SUB.) (Zoology - II)	8.00 to 11.00 am	Nalanda Open University, Patna
31/5/2014	Hindi 100 orUr 50+Hn50	**3.30 to 6.30 pm**	Nalanda Open University, Patna
02/6/2014	(SUB.) (Botany - II)	8.00 to 11.00 am	Nalanda Open University, Patna
02/6/2014	(SUB.) (Physics- II)	8.00 to 11.00 am	Nalanda Open University, Patna
04/6/2014	(SUB.) (Geography -II)	8.00 to 11.00 am	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2014

B.Sc.(Hons.), Part-II

Mathematics (Subsidiary), Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer **Eight** question in all, selecting atleast **One** question from each group. All questions carry equals marks.*

Group - A

1. (a) Evaluate any two of the following integrals :

(a) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (b) $\int \frac{dx}{(1+x)\sqrt{1+3x+x^2}}$ (c) $\int \sqrt{\frac{x}{a-x}}dx$

2. Evaluate the integral as limit of sum :

(a) $\int_a^b \sin x dx$ (b) $\int_a^b \cosh 2x dx$

3. (a) Find the reduction formula for $\int \tan^n x\, dx$.

(b) Evaluate $\lim_{n\to\alpha} \sum \frac{1}{n}\sqrt{\frac{n+r}{n-r}}$

4. (a) Find the length of the arc of the curve $y = \log x$ intercepted between x=1 and x=2.
(b) Find the area enclosed by the curve $xy^2 = a^2(a - x)$.

5. Find the volume and surface of a right circular cone having height 'h' and radius of base a.

6. Solve ant two of the following differential equations :

(i) $y = 2px + p^2y$ (ii) $y = px + \frac{a}{p}$ (iii) $p^2 - 7p + 12 = 0$

7. Find the general solution of the any two following equations :-

(i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ (ii) $\frac{d^2y}{dx^3} + 4y = \sin 2x$ (iii) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x}$

Group - B

8. (a) Find the centre and radius of the circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$, $x - 2y + 2z = 3$.
(b) Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$, represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$.

9. Find the equation of the right circular cylinder whose radius is a and axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$.

Group - C

10. (a) Define a convex set $S \subseteq R^n$ and prove that the intersection of two convex sets is convex.
(b) Show that the set of all convex combinations of a finite number of linearly independent vectors $v_1, v_2, v_3, \ldots\ldots\ldots\, v_n$ is a convex set.

11. Write short notes on the following :-
(a) Interior point (b) Closed set (c) Hyper plane (d) Convex function

Group - D

12. Find the equation of line of action of the resultant of a system of Coplanar forces.

13. Three forces each equal to P, act along the sides of the ΔABC taken in order. Prove that the resultant force has magnitude $P\sqrt{1 - 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$.

Group - E

14. (a) Explain the principle of virtual work and point out its utility.
(b) The speed of a particle moving along x-axis, is given by $v^2 = 4x - x^3$. Show that the acceleration and speed are related by $27v^4 = 8(2 - f)(4 + f)^2$.

15. (a) Explain principle of energy and derive its Mathematical form.
(b) A bullet of mass m is moving with velocity u, strikes a block of mass m which is free to move in the direction of motion of the bullet and is embedded in it. Show that the loss of K.E. due to impact is $\frac{1}{2}\left(\frac{M}{M+m}\right)mu^2$.

16. (a) Define Simple Harmonic Motion (SHM). Derive the displacement in terms of time consumed for SHM.
(b) Find the radial and transverse velocities and accelerations for a particle moving along a plane curve.

Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Honours), Part-III

Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group.

Group 'A'

1. (a) Define a metric space and give a suitable example for it.
   (b) State and establish Canchy's-Swartz Inequality.
2. (a) Let (X, d) be a metric space. Prove that the union of any family of open sets in X, is open.
   (b) Prove that, in a metric space, a set is open, if it is an union of open spheres.
3. (a) Define a derived set. If (X, d) be a metric space and A $\subseteq$ X, then prove that A' (derived set) is closed.
   (b) Prove that every convergent sequence in a metric space, is a Cauchy sequence.
4. A sub set Y of a metric space (X, d) is closed iff the limit of every convergent sequence of points in Y, is in Y.

Group 'B'

5. (a) Let (X, d_1) and (Y, d_2) be two metric spaces and f is a mapping from X into Y. Then f is continuous iff $f(\overline{A}) \subseteq \overline{f(A)}$ for every sub-set of X.
   (b) Let (X, d_1) and (Y, d_2) be two metric spaces and f is a mapping of X into Y. Then show that if f is constant, then f is continuous.
6. Define homeomorphism between two topological spaces. Give an example of one-one continuous mapping of a topological space onto another topological space, such that the mapping is not homeomorphism. Justify your answer.

Group 'C'

7. (a) If f be a bounded function on [a, b] and p_1, p_2 are partitions such that p_1 is finer than p_2, then show that $L(p_1) \leq L(p_2) \leq \cup(p_2) \leq \cup(p_1)$.
   (b) The function f defined on R (set of real numbers) as $f(x) = \begin{cases} 0, \; when\ x\ is\ rational \\ 1, \; when\ x\ is\ irrational \end{cases}$
   Show that f is not integrable on any interval, though it is bounded function.
8. (a) Every monotonic function defined on a closed interval, is integrable in the ense of Riemam. Prove this statement.
   (b) State and prove fundamental theorem of Integral Calculus.

Group 'D'

9. State and establish Canchy integral test.
10. (a) Prove that $\sum_{1}^{\alpha} \frac{1}{n(4n^2 - 1)}$ is convergent and hence deduce that $\sum_{1}^{\alpha} \frac{1}{n(4n^2 - 1)} = \frac{3}{2} - 2 \log 2$.
    (b) Define conditionally convergent series. Prove that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots$ to $\propto$ is conditionally convergent.

Group 'E'

11. (a) Introduce the detail idea of normed linear space.

(b) Prove that the set C^n of all n-tupples $z = (z_1, z_2, ..., z_n)$ of complex numbers under the addition and scalar multiplication defined as $z + w = (z_1 + w_1, z_2 + w_2, ..., z_n + w_n)$ and $\alpha(z_1, z_2, ..., z_n) = (\alpha z_1, \alpha z_2, ..., \alpha z_n)$, forms a linear space.

12. (a) Define continuous linear transformation of a normed linear space N to a linear space N'. Let N be a normed linear space and I be the identity mapping of N to itself. Then show that I is a linear transformation.

(b) What is an inner product function? Give an example of it. Prove that an inner product space is uniformly convex.

Examination Programme-2014
B.Sc (Part–III)
Botany, Chemistry, Mathematics, Physics and Zoology (Honours)

Date	3.30 to 6.30 P.M.	Examination Centre
20/5/2014	Honours Paper–V	Nalanda Open University, Patna
22/5/2014	Honours Paper–VI	Nalanda Open University, Patna
24/5/2014	Honours Paper–VII	Nalanda Open University, Patna
26/5/2014	Honours Paper–VIII	Nalanda Open University, Patna
28/5/2014	Paper –XV (General Studies)	Nalanda Open University, Patna

Nalanda Open University
Annual Examination - 2014
B.Sc. Mathematics (Honours), Part-III
Paper-VI

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group.

Group 'A'

1. (a) Define centre of a group G and prove that, this is a normal sub-group of G.
   (b) Introduce the relation of Conjugacy on a group. Prove that the relation of Conjugacy is an equivalent relation on G.
2. What do you mean by Kernels and Ideals of a ring? Let R = Z_6 (a ring), then show that I = {[0], [3]} is an ideal of R.
3. State and prove the fundamental theorem of homomorphism of rings.
4. (a) Introduce the concept of Quotient field of an integral domain.
   (b) Prove that the ring of integers is a principal ideal ring.

Group 'B'

5. State and prove Schroder-Bernstein theorem.
6. (a) For any three cardinal numbers α, β, γ, prove that :
   (i) $\alpha + \beta = \beta + \alpha$ (ii) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$ (iii) $\alpha.\beta = \beta.\alpha$
   (b) If E is any set. Then card P(E) = 2 Card E, where P(E) is the power set of A.
7. (a) State and prove Zorn's lemma.
   (b) Find out the ordered type of $Q \cap (a, b)$ taken with usual order.

Group 'C'

8. (a) Show that every partition of a set determines an equivalence relation.
   (b) Find the probability distribution of the number of heads when three coins are tossed.
9. (a) Find the mean and variance of Binomial distribution.
   (b) A die is thrown 20 times. Getting a number greater than 4 is considered a success. Find the mean and variance of the number of success.

Group 'D'

10. (a) If ${}^{n}P_{r} = nx\ {}^{n-1}P_{r-1}$, then find x.
    (b) Find the number of solution of the equation $x + y + z = = 15$, x, y, z be non-negative integers.
11. (a) State how generating function can be used to get the number of r combinations from a set of n elements, when repetition of elements is allowed.
    (b) What is the solution of recurrence relation $a_n = 6\ a_{n-1} - 9\ a_{n-2}$
    With initial conditions $a_0 = 1$, $a_1 = 6$
12. (a) Define Continuity and differentiability of a complex valued function. Show that the function $f(z) = |z|^2$ is continuous every where, but no where differentiable except at the origin.
    (b) State Cauchy's integral formula and derive it. Find the simple poles of $f(z) = \dfrac{1}{\sin z - \cos z}$.

Examination Programme-2014
B.Sc (Part–III)
Botany, Chemistry, Mathematics, Physics and Zoology (Honours)

Date	3.30 to 6.30 P.M.	Examination Centre
20/5/2014	Honours Paper–V	Nalanda Open University, Patna
22/5/2014	Honours Paper–VI	Nalanda Open University, Patna
24/5/2014	Honours Paper–VII	Nalanda Open University, Patna
26/5/2014	Honours Paper–VIII	Nalanda Open University, Patna
28/5/2014	Paper –XV (General Studies)	Nalanda Open University, Patna

Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Honours), Part-III

Paper-VII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group.

Group 'A'

1. (a) Define a Hyper-plane in R^n and give its equation.
   (b) What is a Convex set? Prove that the intersection of two convex sets is also a convex set.
2. (a) Give general form of L.P.P (Linear Programming Problem) and state its different constituents.
   (b) Solve the following linear programming problem graphically

   $$\text{Maximize } Z = 5x_1 + 3x_2$$
   $$\text{Subject to constraints } 3x_1 + 5x_2 \le 15$$
   $$5x_1 + 2x_2 \le 10$$

   Where $x_1 \ge 0, x_2 \ge 0$
3. (a) What are slack and surplus variables?
   (b) Obtain all basic solutions of the following system :

   $$x_1 + 2x_2 + x_3 = 4,\ 2x_1 + x_2 + 5x_3 = 5$$
4. Use the simplex method to solve the following linear programming problem :

   $$\text{Max. } Z = 5x_1 + 7x_2$$
   $$\text{Subject to } 2x_1 + 3x_2 < 13;$$
   $$3x_1 + 2x_2 \le 12;\ x_1 > 0, x_2 > 0$$

Group 'B'

5. (a) Solve $(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$
   (b) Give the geometrical interpretation of the equation $Pdx + Qdy + Rdz = 0$ and prove that the locus of $Pdx + Qdy + Rdz = 0$ is orthogonal to the locus of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
6. Solve the simultaneous equations $\frac{dx}{dt} + 2x - 3y = t$, $\frac{dy}{dt} - 3x + 2y = e^{2t}$.
7. Apply Charpit's method to find the complete integral of $(p^2 + q^2)yz = qz$. Also find its general and singular integral.
8. (a) Find the attraction of uniform rod at an external point.
   (b) Define potential of an attracting mass at a point. Hence, find the potential of a circular disc at an interior point of the disc.
9. (a) Derive the formula for the pressure at a point in a heavy homogeneous fluid at rest under gravity.
   (b) Define centre of pressure of a plane area immersed in a fluid. Describe Mathematical method (Integration) of finding centre of pressure of a plane area immersed in a fluid.
10. (a) Find the resultant vertical fluid thrust on the lower half of the curved surface of a cylindrical pipe of length 'h' and radius 'a' when the pipe is full.
    (b) Describe the equilibrium of a floating body under the action of a given system of external forces.
11. A mass of fluid is at rest under the action of forces $X = (y + z)^2 - x^2$, $Y = (z + x)^2 - y^2$, $Z = (x + y)^2 - z^2$ acting per unit mass. Find the equation of surface of equal pressure and curves of equal pressure and density.

Nalanda Open University

Annual Examination - 2014

B.Sc. Mathematics (Honours), Part-III

Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions. All questions carry equal marks. **(Calculator not allowed)**

1. (a) Express $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial notation, interval of differencing being unity.

   (b) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$

2. (a) Find the third divided difference with arguments 2, 4, 9 and 10 of the function $f(x) = x^3 - 2x$

   (b) Prove that following $u_0 + u_1x + u_2x^2 + \ldots\ldots = \frac{u_0}{1-x} + \frac{x\Delta u_0}{(1-x)^2} + \frac{x^2\Delta^2 u_0}{(1-x)^3} + \ldots\ldots\ldots\ldots$.

3. Describe Newton-Gregory formula for forward Interpolation.

4. Find the first and second order derivatives of the function $y = f(x)$ at the point $x = 1.1$ when the values of the function are given as under :

x	1	1.2	1.4	1.6	1.8	2.00
$f(x)$	0.00	0.128	05440	1.2960	2.4320	4.00

5. (a) Derive the general quadrature formula for equidistant values of x.

   (b) Use Euler Maclaurin's formula to evaluate $\int_0^1 \frac{dx}{1+x}$.

6. Describe Simpson's one third rule for numerical integration. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule.

7. (a) Formulate the difference equation for the relation $f(x) = C \cdot 3^x + x \cdot 3^{x-1}$.

   (b) Solve the difference equations :

   (i) $16U_{x+2} - 8U_{x+1} + U_x = 0$ (ii) $U_{x+2} - 4U_{x+1} + 13U_x = 0$

8. (a) Describe the Picard's method of successive approximation for the solution of the differential equation $\frac{dy}{dx} = f(x, y)$.

   (b) Solve the differential equation $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 1$ by Runge-Kutta rule, from $x = 0$ to $x = 0.4$ and $h = 0.1$

9. (a) Solve the system of equation by Gauss-Seidel method :

   $$27x + 6y - z = 85,\ 6x + 15y + 3z = 72,\ x + y + 54z = 110$$

   (b) Explain Relaxation method for finding the solution of a system of linear algebraic equation AX = B.

10. (a) Use Bisection Method to find solution of the equation $x^3 - 9x + 1 = 0$ between $x = 2$ and $x = 4$.

    (b) Give geometrical description of Newton-Raphson's method.