NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–I
(Advanced Abstract Algebra)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a composition series of a group. Prove that every finite group G has a composition series.
   (b) Find all composition series of $Z_5 \times Z_5$.
2. State and prove Schreier's theorem.
3. (a) Prove that an Euclidean ring possesses an unit element.
   (b) Show that in an integral domain D, two non-zero elements a and b in D are associates iff a/b and b/a.
4. (a) Show that in every principal ideal domain, each pair of elements has a greatest common divisor.
   (b) Show that the ring of polynomials over the field of reals, is an Euclidean Ring.
5. (a) Define a submodule of a module M. Show that arbitrary intersection of submodule of module M, is a submodule.
   (b) Show that the module M is the direct sum of two sub-modules M_1 and M_2 iff (i) $M_1 + M_2$ and (ii) $M_1 \cap M_2 = \{0\}$.
6. (a) Prove that in a commutative ring R with unity, an ideal iff the residue class ring R/S is a field.
   (b) Let M be the set of all ordered n-tupples of elements of R, then show that it forms a module over R with suitable operations.
7. State and prove the third theorem of isomorphism.
8. (a) Prove that any unital irreducible R-module is cyclic.
   (b) Let M is a R-module and $\lambda(m) = \{x \in R : xm = 0 \text{ and } m \in M\}$. Show that $\lambda(m)$ is a left ideal of R.
9. What is the concept of field extension ? Prove that the field C of complex numbers is a finite extension of field R of real numbers.
10. (a) If *A(F)* be the collection of all automorphisms of a field F. Then prove that *A(F)* is a group with respect to the operation of composite of maps.
    (b) Find the Galois group of the equation $x^3 - 2 = 0$, over the field *a* of rational numbers.

* * *

Examination Programme, 2014

M.Sc. Mathematics, Part–I

Date	*Paper*	*Time*	*Examination Centre*
05.07.2014	Paper–I	3.30 PM to 6.30 PM	Nalanda Open University, Patna
07.07.2014	Paper–II	3.30 PM to 6.30 PM	Nalanda Open University, Patna
09.07.2014	Paper–III	3.30 PM to 6.30 PM	Nalanda Open University, Patna
11.07.2014	Paper–IV	3.30 PM to 6.30 PM	Nalanda Open University, Patna
15.07.2014	Paper–V	3.30 PM to 6.30 PM	Nalanda Open University, Patna
17.07.2014	Paper–VI	3.30 PM to 6.30 PM	Nalanda Open University, Patna
19.07.2014	Paper–VII	3.30 PM to 6.30 PM	Nalanda Open University, Patna
21.07.2014	Paper–VIII	3.30 PM to 6.30 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–II
(Real Analysis)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Define open sets in R (real line) and prove the following statements :—
   (a) The empty set ϕ and real line R are open sets.
   (b) The union of any family of open sets, is an open set.
   (c) The intersection of finite number of open sets in R is an open set.

2. (a) If *f* and *g* are functions of bounded variation on [a, b], then *f + g* and *f.g* are bounded variation on [a, b]. Establish this statement.
   (b) Let *f* be a function defined by $f(x) = x\,Cos\left(\frac{\pi}{2x}\right),\ if\ x \neq 0,$ $= 0\ ,\ if\ x = 0$ then prove that *f* is continuous on [0, 1] but *f* is not of bounded variation on [0, a].

3. (a) If *P** is a refinement of the partition *P* of [a, b]. Then prove that $L(P, f, \propto) \leq L(P^*, f, \propto)$ and $U(P^*, f, \propto) \leq U(P, f, \propto)$.
   (b) If *f* is continuous on [a, b] then *f* is integrable with respect to $\propto$ on [a, b] in the sense of Riemann-Stieitjes, prove this statement.

4. (a) State and prove the first mean value theorme of R–S integral.
   (b) Prove directly from definition of Stieltjes integral that $\int_a^b d(\propto(x)) = \propto(b) - \propto(a)$.

5. (a) Define Norm of a vector space and give an example of it based on logical proof.
   (b) Introduce the partial derivatives of a vector valued function *f* defined on an open subset *E* of R^n.

6. State and prove mean value theorem for differentiable vector valued function.

7. (a) What do you mean by inform convergence of a power series and its radius of convergence.
   (b) Obtain the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{\lfloor 2n}{\left(\lfloor n\right)^2} x^n$.

8. (a) Discuss Lagarange's Multiplier Method for determining extreme values of a function.
   (b) Find the extreme values of *xy (a – x – y)*.

9. State and prove Implicit function theorem.

10. (a) Define functionally dependence of functions in R^n, extensively.
    (b) Find the Jacobian $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$, where $x = r\,Sin\theta Cos\phi,\ \ y = r\,Sin\theta \cdot Cos\phi,\ \ z = r\,Cos\theta$.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–III
(Measure Theory)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Show that outer measure is countably sub-additive.
   (b) Prove that the class of measurable sets is a σ –ring.
2. (a) Give an example of a set E such that outer measure $m^*(E) = 0$, but E is not countable.
   (b) Prove that Cantor's ternary set is an uncountable set of measure zero.
3. (a) Show that the class of measurable functions is closed with respect to usual analytical operations.
   (b) Let f and g be measurable functions, then show that $f + g$ and $f.g$ are also measurable.
4. (a) Show that if $\{f_n\}$ is a sequence of measurable functions then $\lim\limits_{n \to \infty} f_n$ (if exists), is measurbale.
   (b) Prove that if f is measurable, then for each extended real number a, the set $\{f = a\}$, is a measurable set, but converse is not true.
5. State and prove Egoroft theorem.
6. (a) Establish the additivity property of L–integrable function.
   (b) Show that $\int\limits_{A \cup B}(f) = \int_A (f) + \int_B (f)$ in usual notations, where integration is performed in Lebesgue sense.
7. (a) Examine the L-integrability of $f(x)$ over [0, 1], where $f(x) = \frac{1}{x}; (0 < x \leq 1), f(0) = \infty$.
   (b) Show that if f is measurable function on each of the sets in countable collection $\{E_i\}$ of disjoint measurable sets, then f is measurable on $\bigcup\limits_{i=1}^{\infty} E_i$.
8. State and prove F. Riesz's theorem.
9. (a) State and prove dominated convergence theorem.
   (b) Use above theorem to evaluate $\lim\limits_{n \to \infty} f_n(x)$, where $f_n(x) = \dfrac{n^{1/2} x}{1 + n^2 x^2}$, $\quad 0 \leq x \leq 1$.
10. (a) Show that function of bounded variation over [a, b] is bounded over [a, b], but the converse is not true. Satisfy yourself by means of an example.
    (b) Prove that if f is of bounded variation over [a, b], then f' exists almost everwhere on [a, b].

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–IV
(Topology)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define the boundary of a set in a topological space. Show that a set A of a topological space (X, T) is open *iff* $b(A) = \overline{A} - A$.
   (b) Show that A is open if and only if A is disjoint from its boundary.

2. (a) Define cofinite topology on a nonempty set *X*.
   (b) Prove that the intersection of any two topologies on a not empty set *X* is a topology on *X*.

3. (a) Let *(X, T)* be a topological space, then a subset A is open *iff* A is neighbourhood of each of its points.
   (b) Prove that in a topological space finite union of closed sets is closed.

4. (a) Let A and B be subsets of a topological space X, then show that $b(A \cup B) = b(A) \cup b(B)$.
   (b) Let $X = \{a, b, c, d, e\}$ and topology T is given by $T = \{\phi, X, \{a\}, \{a, b, e\}, \{a, b, c, d\}, \{a, c, d\}\}$ and $A = \{c, d, e\}$. Determine limit point, closure, interior, exterior and boundary of the set A.

5. (a) Define base (open base) and sub-base (open sub-base) of a topological space. Give examples of each of these.
   (b) Prove that the open rectangle in the Euclidean plane form an open base.

6. (a) Define continuity on a topological space. Prove that the mapping *f* of topological space *(X, T)* into an indiscrete space *(Y, I),* is continuous.
   (b) Discuss sequential continuity and its relation with continuity.

7. (a) Show that every compact sub-space of real line is closed and bounded.
   (b) Show that every co-finite space is compact.

8. (a) Prove that a one-to-one continuous mapping of compact space into a Hausdorff space, is a homomorphism.
   (b) Show that the product of any number of non-empty class of Hausdorff space is a Hausdorff space.

9. (a) In a T_1 –space *X*, a point *p* in *X* is an accumulation point of a subset *A* of *X iff* every open set containing *p* contains infinitely many distinct points of *A*, prove this.
   (b) Let $X = \{a, b, c\}$ and $T = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Show that it is a T_0 –space but not a T_1 –space.

10. (a) Prove that a subset *Y* of a topological space *X* is disconnected *iff Y* is the union of two non-emply separated sets.
    (b) Show that any continuous image of a connected space is connected.

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–V

(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that if *V(F)* is an n-dimensional space, then *V* is isomorphic to F^n.
   (b) Show that $B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ forms a basis of R^3.

2. If $V = W_1 \oplus W_2 \oplus --- \oplus W_R$ then show that there exists linear operators $E_1, E_2, ---, E_R$ on V such that, (i) Each E_i is a projection; (ii) $E_iE_j = 0$ if $i \neq j$; (iii) $E_1 + E_2 + ---- + E_R = I$; (iv) Range $E_i = W_i$ and conversely.

3. Define a linear functional from a vector space to its field. Prove that a function *f* on R^n defined by $f(x) = f(x_1, x_2, ---, x_n) = a_1x_1 + a_2x_2 + ---- + a_nx_n$ is a functional on R^n, where $a_1, a_2, ---, a_n$ be fixed scalars in *R*.

4. (a) If *V* be a finite dimensional vector space and *B* be a basis of *V* and *B′* be dual basis for *V*, then show that $B'' = (B')' = B$.
   (b) Let $V_3(R)$ be a vector space and $B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ be a basis of $V_3(R)$. Find its dual basis *B′*.

5. (a) Define a bilinear form on a vector space $V(K)$. Show that $b(x, y) = [x_1, x_2]\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ is a real bilinear form.
   (b) Let *T* be a linear operator on *V(F)* and *f* be a bilinear form on *V*. Then show that a function $g : V \times V \to K$ defined as $g(\alpha, \beta) = f(T\alpha, T\beta)$ is a bilinear form on *V*.

6. (a) Define lattice and sub-lattice and makeout their difference with examples.
   (b) What is a modular lattice ? Prove that the set *L* of all ideals of a ring, is a modular lattice.

7. State and establish Sylvester's law of inertia.

8. (a) Define a Boolean algebra an give an example of it.
   (b) Prove that the intersection of any two Sub-Algebra of a Boolean algebra *B*, is a Boolean Sub-Algebra of *B*.

9. (a) Prove that if *B* is Boolean Algebra and $x, y, z \in B$, then $x \wedge (y - Z) = (x \wedge y) - (x \wedge Z)$.
   (b) Prove that a complemented distributive lattice is a Boolean Algebra.

10. (a) Define similar linear transformation on a finite dimensional vector space $V(K)$. If T_1 and T_2 are similar linear transformation, then show that $\det T_1 = \det T_2$.
    (b) Reduce $A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ to a triangular matrix by finding an invertible matrix.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–VI
(Complex Analysis)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define analytic function. If $f(z) = u + iv$ be an analytic function in a domain *D*. Prove that the curves u = constant and v = constant form two orthogonal families.

   (b) Show that $u = x^3 - 3xy^2$ is a harmonic function. Find the corresponding analytic function.

2. What is radius of convergence of a power series ? Discuss the three possible case for the vanishing of radius of convergence.

3. (a) By giving the idea of Bilinear transformation, prove that the set of all Bilinear Transformations under their product forms a group.

   (b) Find the condition that the transformation $w = \dfrac{az + b}{cz + d}$ transforms the unit circle in w-plane into a straight line of z-plane.

4. Describe each of the mappings geometrically given as under (i) $w = z^n$, (ii) $w = z^2$ and (iii) the inverse mapping of $w = z^{½}$.

5. State and establish Cauchy's integral formula.

6. (a) If *f(z)* is an analytic function within and an a circle C given by $|z - a| = R$ and if $|f(z)| \le M$ for every z on C, then prove that $|f^n(a)| \le \dfrac{M\underline{|n}}{R^n}$.

   (b) Evaluate $\int_C \dfrac{e^{2z}dz}{(z + 1)^4}$, where C is the circle $|z| = 3$.

7. State and prove Laurent's theorem.

8. Find the Laurrent's expansion of $\dfrac{Z}{(z + 1)(z + 2)}$ about singularity $z = -2$. Specify the region of convergence and nature of singularity at $z = -2$.

9. (a) What do you mean by poles and residues, with reference of complex valued function ?

   (b) Find the poles and residues at $z = 4$ and $z = 5$.

10. (a) State and prove Cauchy's Residue theorem.

    (b) Using Cauchy residue theorem, evaluate $\int_C \dfrac{e^z dz}{z(z - 1)^2}$, where C is the circle $|z| = 2$.

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–VII

(Theory of Differential Equations)

Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. State and prove Cauchy-Peano Existence theorem.

2. State and prove Ascoli's Lemma.

3. Define a linear system and show that it satisfies Lipsehitz condition, and set of solutions forms a vector space.

4. (a) Show that the following function doesnot satisfy the Lipschitz condition in the region indicated $f(x, y) = \frac{e^x}{y^2}, f(x, 0) = 0, |y| \le \frac{1}{2}, |x| \le 2$.

   (b) Compute the first three successive approximations for the solution of the Initial Value Problem (IVP) $y' = y^2, y(0) = 1$.

5. (a) $A(x)$ be continuous on [a, b]. Then show that the IVP, $\frac{dy}{dx} = A(x)y, y(r) = s, a \le r \le b$, $|s| < \propto$ has a unique solution on [a, b].

   (b) Solve the system of linear equations $y_1' = 2y_1 + y_2, y_2^1 = 3y_1 + 4y_2$.

6. (a) Discuss linear homogeneous system and fundamental matrix.

   (b) Show that the set of all solutions of Linear Homogeneous System of n^{th} order on an interval *I*, is a complex vector space of dimension *n*.

7. Define stable and asymptotically stable solutions. Let $x' = Ax + f(t, x)$, where A is a real constant matrix with characteristic roots all having negative real parts. If *f* be real, continuous for $|x|$ and $t \ge 0$ and $f(t, x) = 0, [|x| \to 0]$ uniformally in $t, t \ge 0$. Then show that the indentically zero solution is asymptotically stable.

8. (a) Find the nature of the critical point (0, 0) of the system $\frac{dx}{dt} = 3x + 4y, \frac{dy}{dt} = 3x + 2y$.

   (b) Find the type and stability of the critical point (0, 0) of the non-linear system $\frac{dx}{dt} = x + 4y - x^2, \frac{dy}{dt} = 6x - y + 2xy$.

9. (a) What is the meaning of Generating function for Legendre polynomial ? Hence find it.

   (b) Describe orthogonal property of Laguerre polynomial.

10. Find the series solution of Bessel's Differential Equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0$.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–I, PAPER–VIII
(Set Theory, Graph Theory, Number Theory, Differential Geometry)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define countable set. Prove that the interval [0, 1] is uncountable.
   (b) If A and B are two countable sets, then show that A × B is also countable.

2. (a) For any cardinal numbers $\alpha,\ \beta,\ \gamma$; show that $\alpha^{\beta\gamma} = \left(\alpha^{\beta}\right)^{\gamma}$.
   (b) Prove that the axioms of choice implies Zorn's Lemma.

3. (a) Explain the homomorphism between two graphs. Give two examples of it.
   (b) If G is a connected planar simple graph, then prove that G has a vertex of degree not exceeding five.

4. (a) Prove that an undirected graph is a tree iff there is unique path between any two vertices.
   (b) If a tree has n vertices of degree 4. Find the value of n.

5. (a) State and prove Euler's theorem.
   (b) Express $\left(2^2+5^2\right)\left(3^2+8^2\right)\left(4^2+7^2\right)$ as sum of two squares.

6. (a) If $a, b \in Z$, then prove that (a, b) exists and is unique. Also establish that there exists integers s and t such that $(a, b) = as + bt$.
   (b) Find (24, 63) as a linear combination of 24 and 63.

7. (a) State and prove Chinese reminder theorem.
   (b) Find the general solution of the equation $8x + 5y = 81$.

8. (a) Define osculating plane and derive its scalar and vector equations at a point $P(\vec{r})$ on the curve.
   (b) Find the curvature and torsion for the curve $x = a\,Cos\,t,\ y = a\,Sin\,t,\ z = c\,t$.

9. (a) Explain associated Bertrand Curve and derive the result $TT_1 = \dfrac{Sin^2\alpha}{a^2}$ and $\left(1 - aR\right)\left(1 + aR_1\right) = Cos^2\alpha$, where the quantities have their usual meanings.
   (b) Find the involutes and evaluates of the circular helix $\vec{r} = \left(a\,Cos\,\theta,\ a\,Sin\,\theta,\ a\,\theta\tan\alpha\right)$.

10. (a) Explain about asymptotic lines and prove that the principal directions bisect angles between asymptotic directions.
    (b) Prove that the surface $xy = (Z - C)^2$ is a developable surface.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–IX
(Numerical Analysis)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Derive Newton's forward interpolation formula.
   (b) Estimate population for the year 1905 by using Newton's formula for interpolation.

Year :	1891	1901	1911	1921	1931
Population:	98,752	1,32,285	1,68,076	1,95,690	2,46,050

2. (a) Describe the two methods of representing any given polynomial in factorial notation.
   (b) Evaluate $\Delta^n\left(\frac{1}{x}\right)$.

3. (a) The equation $x^6 - x^4 - x^3 - 1 = 0$ has one root between 1.4 and 1.5. Find the root to four places of decimal by Ragula-Falsi method.
   (b) Describe iteration method and use it to find a real root of the equation $f(x) = x^3 + x^2 - 1 = 0$.

4. (a) Explain Newton-Raphson's method geometrically and discuss its failure cases.
   (b) Apply Newton-Raphson's method to find the root of $x^4 - x - 10 = 0$ which is nearer to $x = 2$ correct to three decimal palces.

5. (a) Use Gauss forward formula to find y_{30}, where $y_{31} = 18.4708$, $y_{25} = 17.8144$, $y_{29} = 17.1070$, $y_{33} = 16.3432$, $y_{37} = 15.5154$.
   (b) Employ Lagrange's interpolation formula, to find the form of the function *f(x)* from the table given below :–

x	0	1	3	4
y	–12	0	12	24

6. (a) Prove that the value of the divided difference is independent of the order of the argument i.e. the divided differences are symmetric functions of their arguments.
   (b) Find the polynomial of the lowest degree which assumes the values 3, 12, 15, –21 when *x* has the values 3, 2, 1, –1 respectively (use Newton interpolation divided difference formula).

7. (a) Find the first order derivative of the function formed on the table given below, at the point *x = 1.2*.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

   (b) Using divided differences, find the value of *f(8)*, given that *f(6)* = 1.556, *f(7)* = 1.690, *f(9)* = 1.908, *f(12) = 2.158*.

8. (a) Find from the following table, the area bounded by the curve and the ordinates x = 7.47 and x = 7.52
   (b) Find $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule. Hence, obtain an approximate value of π.

9. (a) Explain order and degree of a difference equation. Form the difference equation corresponding to the family of Curves $y_x = ax^2 + bx - 3$.
   (b) Show that $y_x = C_1 + C_2 2^x - x$ is a solution of the difference equation $y_{x+2} - 3y_{x+1} + 2y_x = 1$.

10. Define linear homogeneous and non-homogeneous difference equations. Explain the method of construction of general solution of a non-homogeneous difference equation.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–X
(Functional Analysis)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Consider the linear space of all bounded sequences $x = (x_1, x_2, x_3, ---, x_n, ---)$ of scalars. Define $\| x \|_\infty = \underset{n}{Sup} |x_n|$ and denote this space by l_∞. Show that this space is a Banach space.

2. Prove that the space $e[0, 1]$ of all complex valued functions on [0, 1] is not a Banach space, with respect to the norm given by $\| f \| = \int_0^1 |f(t)dt$.

3. (a) Define linear functional and give its an example with justification.

   (b) Produce an example of a normed linear space which is not a Banach space with justification.

4. (a) Show that any two normal linear spaces having the same finite dimension are homeomorphic.

   (b) Show that a normed linear space *N* can be embedded into *N* **.

5. If $\| x \|$ and $\| x \|^1$ generate the same topology on a linear space *L*, then show that these norms are equivalent.

6. (a) If *x* and *y* are Banach spaces and if *T* is a continuous linear transformation of *x* to *y*. Then, show that *T* is an open mapping.

   (b) State and prove closed graph theorem.

7. (a) State polarization identity and explain about it in an inner product space.

   (b) Introduce the concept of orthogonal complements in an inner product space and derive Pythogorus theorem from it.

8. (a) If *x* and *y* are any two vectors in an inner product space, then prove that $|(x, y)| \le \| x \| \cdot \| y \|$. Also prove that the equality holds iff *x* and *y* are linearly independent.

   (b) Show that the inner product space is jointly continuous.

9. After defining conjugate operator *T** of an operator *T* on a Hilbert space *H*, prove the following properties (i) $(\alpha T)^* = \overline{\alpha}\, T^*$, (ii) $T^{**} = T$, (iii) $\| T^* \| = \| T \|$ *and* $\| T^* T \| = \| T \|^2$.

10. (a) Prove that every Hilbert space is reflexive.

    (b) Apply the Gram-Schmidt process to the vectors $x_1 = (1, 0, 1)$, $x_2 = (1, 0, -1)$, $x_3 = (0, 3, 4)$ to obtain an orthogonal basis of R^3 with the standard inner product.

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XI

(Partial Differential Equations)

Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Explain Cauchy's problem of first order partial differential equation.
   (b) Solve the Cauchy problem $zp + q = 1$, where the initial data curve is $x_0 = \mu,\ y_0 = \mu$, $z_0 = ½\mu,\ 0 \le \mu \le 1$.
2. (a) Find the integral surface of the linear partial differential equation $x\left(y^2 + z\right)p - y\left(x^2 + z\right)q = \left(x^2 - y^2\right)z$ which contains the straight line $x + y = 0,\ z = 1$.
   (b) Construct orthogonal surfaces of the given surfaces $f(x, y, z) = c$.
3. Solve the following partial differential equations,
   (a) $\dfrac{\partial^2 t}{\partial x^2} + \dfrac{\partial^2 t}{\partial y^2} = Cos\,mx\ Cos\,ny$ (b) $(D - D' - 1)(D - D' - 2)z = Sin(2x + 3y)$
4. Describe the working rule for finding C.F. (Complementary function) of reducible non-homogeneous linear partial differential equation with constant coefficients.
5. Solve any two of the following differential equations :—
   (a) $2yq + y^2 t = 1$ (b) $t + s + q = 0$ (c) $s - t = x^2/y^2$
6. Illustrate the method of separation of variables to solve the equation $\alpha^2 \dfrac{\partial^2 y}{\partial x^2} = \dfrac{\partial^2 y}{\partial t^2}$, where $\alpha^2 = \dfrac{T}{\rho}$ satisfying the boundary conditions $y(0, t) = 0,\ 0 \le t \le \propto\ y(l, t) = 0,\ 0 \le t < \propto$. At $t = 0$, the string is released from the initial position $y(x, 0) = f(x),\ 0 \le x \le l,\ \dfrac{\partial y}{\partial t}(x, 0) = g(x),\ 0 \le x \le l$.
7. A string is stretched between two fixed points at a distance 'l' apart. Motion is started by displacing the string in the form $y = y_0\ Sin\left(\dfrac{\pi x}{l}\right)$ by releasing at time t = 0. Find the displacement at any point at a distance *x* from one end at time t.
8. (a) Define equipotential surfaces. Show that the cylinders $x^2 + y^2 = 2\lambda x$ are a possible set of equipotential surfaces in empty space, but the spheres $x^2 + y^2 + z^2 = 2\lambda x$ are equipotential in real state.
   (b) If *V* be the potential of an attracting system at any point $P(x, y, z)$ which doesnot Coincide with any one of the attracting particles, then prove that $\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y} + \dfrac{\partial^2 v}{\partial z^2} = 0$.
9. Find the temperature distribution on inside a square plate of a side having boundary conditions $u(0, y, t) = u(a, y, t) = u(x, 0, t) = u(x, a, t) = 0$ and initial condition $u(x, y, 0) = Cos\dfrac{\pi(x - y)}{a} - Cos\dfrac{\pi(x + y)}{a}$.
10. (a) Find the general solution of heat equation when both ends of a bar are kept at zero temperature and the initial temperature is given.
    (b) Obtain the general solution of wave equation $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$, given that initial deflection $u(x, 0) = f(x)$ and initial velocity $\left(\dfrac{\partial u}{\partial t}\right)_{t=0} = g(x)$.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–XII
(Analytical Dynamics)
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that in a simple dynamic system, T + V = constant where T & V have their usual meaning.

   (b) Explain the term Holonomic and Non-holonomic dynamical system by giving suitable examples.

2. (a) Derive Lagrange's equation of impulsive motion in a holonomic dynamical system.

   (b) A bead is sliding on a uniformly rotating wire in a force free space. Derive the equation of motion.

3. (a) Construct Routhian function and Routh's equation for the solution of a problem involving cyclic and non-cyclic co-ordinates.

   (b) Using Routhian equation of motion to determine the motion of a uniform heavy rod turning about one end which is fixed.

4. (a) State the principle of least action and hence establish it in terms of arc length of a particle-path.

   (b) A particle moves in a plane under a central force depending on its distance from the origin. Construct the Hamiltonian of the system and derive Hamilton's equation of motion.

5. (a) Discuss small oscillations, normal co-ordinates and normal mode of vibration.

   (b) Describe the effect of constraints on the period of normal oscillations of a dynamical system about a stable equilibrium position.

6. (a) Define the generating function of a transformation and give an example of a generating function of a transformation.

   (b) Show that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\,Cot\,p$, is canonical.

7. Define Poisson's Bracket and show that the Poisson's Bracket obeys the distributive laws i.e.

   (i) $[u + v, w]_{q_r,\,p_r} = [u, w]_{q_r,\,p_r} + [v, w]_{q_r,\,p_r}$

   (ii) $[uv, w]_{q_r,\,p_r} = u[v, w]_{q_r,\,p_r} + [u, w]v_{q_r,\,p_r}$

8. State and prove Jacobi-Poisson theorem.

9. (a) State and prove Jacobi theorem.

   (b) A particle of mass m moves in a force field whose potential in spherical co-ordinates is given by $V = \frac{\lambda\,Cos\,\theta}{r^2}$. Write the Hamilton Jacobi equation and derive the complete solution.

10. (a) Determine the kinetic energy and moment of momentum of a rigid body rotating about a fixed axis.

    (b) Describe the motion of a particle about revolving axes.

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XIII

(Fluid Mechanics)

Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Derive the equation of continuity in spherical polar co-ordinates.

   (b) Determine the stream lines and paths of the particles whose velocity components are $u = \frac{x}{1+t}, v = \frac{y}{1+t}, w = \frac{z}{1+t}$.

2. (a) Define boundary surface and find the condition that $f(x, y, z, t) = 0$ may be a boundary surface.

   (b) Show that $u = -\frac{2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}$ *and* $w = \frac{y}{x^2+y^2}$ satisfies an irrotational motion.

3. (a) State and prove Kelvin circulation theorem.

   (b) A velocity field is given by $\vec{q} = \frac{-\hat{i}y + \hat{j}x}{x^2+y^2}$. Determine whether the flow is irrotational ? Also calculate the circulation round a unit circle with centre at the origin.

4. (a) Derive pressure equation.

   (b) Prove that if the motion of an ideal fluid, for which density is a function of presence p only, is steady and the external forces are conservative, then there exists a family of surfaces which contain the stream lines and vortex lines.

5. (a) What is stream function ? Give its physical significance.

   (b) Show that in two dimensional irrotational motion, stream function velocity potential satisfy Laplace's equation.

6. (a) Make out difference between source and sink. Find the complex potential due to a source of strength m placed at the origin.

   (b) Show that $u = 2Axy, v = A(a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Determine the stream function of fluid motion.

7. (a) Discuss the motion in the case of liquid streaming past a fixed circular cylinder of radius 'a' with velocity U.

   (b) The centre of a circular cylinder of radius a is moving with the velocity U along the x-axis through the fluid, which is at rest, at $\propto$. Show that the co-ordinate (x, y) of a fluid particle satisfy the equations $\frac{dx}{dt} = \frac{Ua^2}{r^2}Cos2\theta, \frac{dy}{dt} = \frac{Ua^2}{r^2}Sin2\theta$, where $x = Ut = rCos\theta, y = rSin\theta$.

8. Derive the equations of motion of a sphere in an infinite mass of liquid at rest at infinity.

9. (a) Define stress tensor and strain tensor and develop the relation between stress tensor and rate of strain tensor.

   (b) The stress tensor at a point P is $\sigma_{ij} = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{bmatrix}$. Determine the stress vector on the plane though P whose unit normal is $\hat{n} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$.

10. (a) Derive Navier-Stokes equation of motion of viscous fluid.

    (b) Use Navier-Stokes theorem to find the vorticity equation.

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XIV

(Operations Research)

Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define hyper plane and hyper sphere. Prove that every hyper plane in R^n is a convex set.
   (b) An animal fodder company needs to produce 500 kg of a mixture having components A and B cost Rs. 5 and 6 per kg respectively. The ingredient A should not exceed 90 kg and B must not be below 70 kg. Find the minimum cost of the mixture.

2. (a) Find basic feasible solution of the system $2x_1 + x_2 + 4x_3 = 11,\ 3x_1 + x_2 + 5x_3 = 14$.
   (b) Reduce feasible solution $x_1 = 2, x_2 = 4$ *and* $x_3 = 1$ of the system $2x_1 - x_2 + 2x_3 = 2$ and $x_1 + 4x_2 = 18$ to a basic feasible solution and mention its kind (degenerate or non-degenerate).

3. (a) Introduce the concept of slack and surplus variables.
   (b) Solve graphically the L.P.P. minimize $z = 5x_1 + 3x_2$
   Subject to $x_1 + x_2 \le 6,\ 2x_1 + 3x_2 \ge 6,\ 0 \le x_1 \le 4,\ 0 \le x_2 \le 3$ *and* $x_1 \ge 0, x_2 \ge 0$.

4. Solve the L.P.P. by simplex method :—
   Maximize $z = 3x_1 + 4x_2$
   Subject to : $x_1 - x_2 \le 1,\ -x_1 + x_2 \le 2,\ x_1 \ge 0,\ x_2 \ge 0$.

5. Use two phase simplex method of solve the L.P.P.
   Maximize $z = 5x_1 - 4x_2 + 3x_3$
   Subject to : $2x_1 + x_2 - 6x_3 = 20,\ 6x_1 + 5x_2 + 10x_3 \le 76,\ 8x_1 - 3x_2 + 6x_3 \le 50$ *and* $x_i \ge 0\ (i = 1,2,3)$

6. Construct the dual problem of the L.P.P.
   Maximize $z = 3x_1 + x_2 + 2x_3 - x_4$
   Subject to : $2x_1 - x_2 + 3x_3 + x_4 = 1,\quad x_1 + x_2 - x_3 + x_4 = 3$ and $x_1 \ge 0, x_2 \ge 0, x_3$ and x_4 are unrestricted.

7. Describe dual simplex method by elaborating every step.

8. For the L.P.P. Maximize $z = 3x_1 + 4x_2 + x_3 + 7x_4$
   Subject to : $8x_1 + 3x_2 + 4x_3 + x_4 \le 7,\ 2x_1 + 6x_2 + x_3 + 5x_4 \le 3,\ x_1 + 4x_2 + 5x_3 + 2x_4 \le 8$ and $x_i \ge 0\ (i = 1, 2, 3, 4)$. Describe the effect of discrete change in a_{ij} (an element of coefficient matrix)

9. (a) Explain minimax and maximini principle in the game theory.
   (b) In a game of maching coins with two players A and B, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and looses ½ unit of value when these is one head and one tail. Determine the pay off matrix, the best strategy for each player and the value of game.

10. (a) Obtain the feasible solution of the N.L.P.P. :—
    Maximize $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
    Subject to : $x_2 \le 8,\ x_1 + x_2 \le 10$ *and* $x_1, x_2 \ge 0$.
    (b) Use Lagrange's multiplier method to solve the non-linear programming problem :—
    $z = ax_1^2 + bx_2^2 + cx_3^2$, where $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 1$.

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XV

(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define inner and outer product of two tensors and prove that the outer product of two tensors is a tensor of rank equal to the sum of ranks of the two tensors.
   (b) Show that any linear combination of tensors of the type *(r, s)* is a tensor of the type *(r, s)*.
2. (a) Introduce the concept of Christofell symbols and prove that $[ij, k] + [jk, i] = \frac{\partial g_{ik}}{\partial x^j}$.
   (b) Derive the law of transformation of Christofel symbols of second kind.
3. (a) State and prove convolution theorem on inverse Laplace transform.
   (b) Find the Laplace transform of (i) $Sinh^2 at$, (ii) $\frac{e^{at} - 1}{a}$ under the condition to be specified by you.
4. (a) Apply Laplace transform to solve $(D^3 - 2D^2 + 5D)y = 0$, if $y = 0, \frac{dy}{dt} = 1$ at $t = 0$ and $y = 1$ at $t = \frac{\pi}{8}$.
   (b) Explain Fourier transform. Find the Fourier transform of $f(x) = \begin{cases} x, & |x| \le a \\ 0, & |x| > a \end{cases}$.
5. (a) Explain about the Fredholm Integral equations of three kinds.
   (b) Solve a Fredholm Integral equation of second kind by successive substitution.
6. (a) Prove that the function $u(x) = (1 + x^2)^{-3/2}$ is a solution of Volterra integral equation $u(x) = \frac{1}{1 + x^2} - \int_0^x \frac{t}{1 + x^2} u(t) dt$.
   (b) Form a Volterra integral equation corresponding to the differential equation given by $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with initial conditions $y(0) = 1, \ y'(0) = 0$.
7. Assume the present value of one rupee to be spent in a year's time is Re 0.90 and C = Rs 3000 (purchase price of the item to be replaced). Capital of the equipment and the running costs are given below :—

Years	1	2	3	4	5	6	7
Running Costs (in Rs.)	500	600	800	1000	1300	1600	2000

8. A readymade garment manufacturer has to process 7 items through two stages of production say cutting and sewing. The time taken for each of these items at the different stages is given below in appropriate units.

Item	1	2	3	4	5	6	7
Cutting Time	5	7	3	4	6	7	12
Sewing Time	2	6	7	5	9	5	8

   Find an order in which these items are to be processed through the stages mentioned above so as to minimize the total processing time.
9. Discuss the deterministic model with instantaneous production (shortage not allowed).
10. Assume that the good trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can handle trains at a time (there being 10 lines, one of which is being reserved for shunting purposes). Calculate the probability that the yard is empty and find average queue length.

* * *

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–XVI
(Programming in 'C')
Annual Examination, 2014

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. What is an Operator? Describe different types of operators that are included in C with examples.
2. What is an expression? What are its components?
3. What is the purpose of the switch statement? How does this statement differ from the other statements?
4. What is an Array? How does an Array differ from an ordinary variable?
5. Write a program to multiply 3x3 matrices in C.
6. What is a function? State three advantages of using functions. What is the purpose of return statement?
7. What is recursion? Write a program to find the roots of a quadratic equation.
8. What is meant by function call? From what part of a program can a function be called?
9. Write a program in C to find Factorial of a given number.
10. Write short notes on any ***Two*** of the following :—
    (a) Increment & Decrement operator
    (b) Pre-Processor Directives
    (c) Mathematical Function

* * *

M.Sc. Mathematics, Part–II, Paper–XVI (Practical)
Counselling & Examination Programme, 2014

Practical Counselling Programme

Enrollment No.	*Date*	*Time*	*Venue*
All Old & New Students	02.09.2014 to 06.09.2014	01:00 PM to 05:00 PM	Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001

Practical Examination Programme

Enrollment No.	*Date*	*Time*	*Venue*
120290001 to 120290259	08.09.2014	12.30 PM to 2.30 PM	Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
120290260 to 120290398 & All Old Students	08.09.2014	3.00 PM to 5.00 PM	

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XVI

(Programming in 'C') – Practical

Annual Examination, 2014

Time : 2 Hours. **<u>SET–I</u>** **Full Marks : 20**

Answer any Two Questions. All questions carry equal marks.

1. Write a C program to generate a Fibonacci series.
2. Write a C program to find out the factorial of entered number using recursive functions.
3. Write a C program to solve a quadratic equation.
4. Write a C program to find the sum of the digits of the entered number.

* * *

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XVI

(Programming in 'C') – Practical

Annual Examination, 2014

Time : 2 Hours. **<u>SET–II</u>** **Full Marks : 20**

Answer any Two Questions. All questions carry equal marks.

1. Write a C program to generate a series of first 10 even numbers.
2. Write a C program to find out the largest of three numbers.
3. Write a C program to solve a quadratic equation.
4. Write a C program to find the reverse of the digits of the entered number.

* * *