NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–I

(Advanced Abstract Algebra)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a composition series of a group. Prove that every finite group G has a composition series.
   (b) Find all composition series of $Z_5 \times Z_5$.
2. State and prove Jordan-Holder theorem for finite group.
3. (a) If D is an integral domain and $a, b, c \in D$, then show that
   (i) a/b *and* $a/c \Rightarrow a/b+c$ (ii) $a/b \Rightarrow a/bx, \; \forall \; x \in D$
   (b) In an integral domain D two non-zero elements $a, b \in D$ are associates iff a/b and b/a.
4. (a) Show that in every principal ideal domain, each pair of elements has a greatest common divisor.
   (b) Show that the ring of polynomials over the field of reals, is an Euclidean ring.
5. (a) Prove that the linear sum of two sub-modules of a R-module M, is also a sub-module of M.
   (b) Let M and N be modules over R and f : M $\rightarrow$ N be a homomorphism. Show that the kernel of *f* is a sub-module of M.
6. State and prove the second theorem of isomorphism on modules.
7. (a) Let $a \in K$ be an algebraic over F, where K is an extension field over F. Then, prove that any two minimal monic polynomials over F must be equal.
   (b) Let a $\in$ K, be an algebraic over F and *P(x)* be a minimal polynomial for 'a' over F. Then prove that *P(x)* is irreducible over F.
8. Let R be the field of real numbers and Q be the field of rational numbers. In R, $\sqrt{2}$ and $\sqrt{3}$ are both algebraic over R. Exhibit a polynomial of degree 4 over Q satisfied by $\sqrt{2} + \sqrt{3}$. Also
   (a) Show that $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$
   (b) Find the degree of $\sqrt{2} + \sqrt{3}$ over Q.
9. Determine the splitting field K of polynomial $x^4 - x^2 + 1$ over Q (the field of rational numbers). Also determine the Galois's group of K over Q. Show that the group so formed is not cyclic.
10. State and prove the fundamental theorem of Galois's theory.

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Revised Examination Programme-2012

M.Sc. Mathematics (Part–I)

Date	*Papers*	*Time*	*Examination Centre*
09.05.2012	Paper–I	3.30 PM to 6.30 PM	Nalanda Open University, Patna
11.05.2012	Paper–II	3.30 PM to 6.30 PM	Nalanda Open University, Patna
15.05.2012	Paper–III	3.30 PM to 6.30 PM	Nalanda Open University, Patna
17.05.2012	Paper–IV	3.30 PM to 6.30 PM	Nalanda Open University, Patna
19.05.2012	Paper–V	3.30 PM to 6.30 PM	Nalanda Open University, Patna
21.05.2012	Paper–VI	3.30 PM to 6.30 PM	Nalanda Open University, Patna
23.05.2012	Paper–VII	3.30 PM to 6.30 PM	Nalanda Open University, Patna
25.05.2012	Paper–VIII	3.30 PM to 6.30 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–II

(Real Analysis)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) State and prove Bolzano-Weirstrars Theorem.
   (b) Define bounded variation function. Show that a monotonic function *f* on [a, b], is of bounded variation on [a, b] and $V(f) = |f(b) - f(a)|$.
2. (a) Let *f* and *g* are of bounded variation on [a, b], then prove that *f* + *g* and *f* •*g* are also of bounded variation.
   (b) If (I_n) is a sequence of closed nested intervals in *R* then show that $\bigcap_{n=1}^{\infty} I_n$ consists of exactly one point.
3. (a) Let $f \in R(\alpha)$ on [a, b], then show that for any constant *C*, the function $Cf \in R(\alpha)$.
   (b) State and prove the first mean value theorem for Riemann-Stieltjes integral.
4. (a) If f is integrable with respect to α in the sense of Riemann-Stieltjes over [a, b] iff for every $\in > o$, there exists a partition of [a, b] such that $\cup(P, f, \alpha) - L(P, f, \alpha) < \in$. Prove this statement.
   (b) If $f(x) = x$ and $\alpha(x) = x^2$. Test the existence of $\int_0^1 f d\alpha$.
5. (a) If $f(x) = \|x\|^2$, prove that $f(c, u) = 2c.u$.
   (b) State and prove mean value theorem for vector-value function from $R^n \to R^m$.
6. (a) Let *E* be an open sub-set of R^n and *f* be a vector-value function $f : E \to R^m$. If *f* is differentiable at a point *c* of *E*, then show that *f* is continuous at *c*.
   (b) Let $f : R^2 \to R$ is defined by $f(x, y) = \frac{xy\,(x^2 - y^2)}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Then show that $D_{12}\, f(0, 0) \neq D_{21}\, f(0, 0)$.
7. (a) State and prove Abel's theorem.
   (b) Obtain the stationary points and stationary value of $u = a^3x^3 + b^3y^2 + c^3z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.
8. State and prove Tauber's (First) Theorem.
9. (a) Describe the applicability of implicet function theorem to the equation $y^2 + 2x^2y + x^5 = 0$, for the existence of unique implicit function near the point (–1, 1).
   (b) Find the first order derivatives of the solutions of *xy sinx* + *cosy* = *0*, near $\left(0, \pi/2\right)$.
   [use implicit function theorem]
10. Let *f* be a function defined on a set $E \subseteq R^n$ with valves in R^n, then show that the components $f_1, f_2, -----, f_n$ of *f* are functionally dependent on *E* iff *f(E)* is no where dense in R^n.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–III

(Measure Theory)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) If E_1 and E_2 are measurable sets, then (i) $E_1 - E_2$ is measurable and (ii) if $E_1 \supseteq E_2$ and $m(E_2) < \infty$, then $m(E_1 - E_2) = m(E_1) - m(E_2)$.
   (b) Let (E_i) be a sequence of pair wise disjoint measurable sets, then show that $m\left(\bigcup_i E_i\right) = \sum_i m(E_i)$.
2. (a) Prove that the class of measurable sets is a σ–ring.
   (b) Show that an enumerable set is measurable with measure zero.
3. (a) A necessary and sufficient condition for a function *f* to be measurable, is that it is the limit of convergent sequence of simple functions.
   (b) Show that the statements (i) $\{f > a\}$ is measurable for each real a and (ii) $\{f < a\}$ is measurable for each real a, are equivalent.
4. State and prove Egoroff's theorem.
5. (a) Prove that every function *f* which is *R*–integrable on $[a, b]$ is also *L* integrable on $[a, b]$ and $R\int_a^b f(x)\,dx = L\int_a^b f(x)\,dx$.
   (b) Examine the *L*–integrability of $\frac{d}{dx}\left(x^2 \sin\frac{1}{x^2}\right)$ over [0, 1].
6. (a) Establish the additivity property of L-integrable function.
   (b) If $|f| \le g$ and g is integrable (L), then show that $|f|$ is integrable (L).
7. (a) State and prove Fatou's Lemma.
   (b) Verify bounded convergence theorem for $f_n(x) = \frac{nx}{1+n^2x^2}$ $(0 \le x \le 1;\ n = 1, 2, 3, ---)$.
8. (a) State and prove Dominated Convergence theorem.
   (b) Use dominated convergence theorem to evaluate $\lim_{n\to\infty}\int_0^1 f_n(x)\,dx$, where $f_n(x) = \frac{xn^{3/2}}{1+n^2x^2}$, $0 \le x \le 1;\ n = 1, 2, 3, ----$.
9. (a) Define absolute continuity. Prove that if *f* is absolutely continuous on [a, b], then it is of bounded variation on [a, b].
   (b) If *f* is absolutely continuous on [a, b] and $f'(x) = 0$ almost every where in [a, b], then confirm that *f* is constant in [a, b].
10. (a) Show that a function may be of bounded variation without being continuous on an interval [a, b] (support your answer by means of suitable example).
    (b) Show that a continuous function need not be an integral (substantiate by a suitable example).

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–: आवश्यक सूचना :–

M.Sc. Mathematics, Part–I के सभी परीक्षार्थियों को सूचित किया जाता है कि पटना नगर निगम चुनाव के कारण दिनांक 17.05.2012 को होने वाली **Paper–IV** की परीक्षा अब दिनांक 18.05.2012 को संध्या 3.30 बजे से 6.30 बजे के बीच आयोजित की जायेगी । अन्य पत्रों की परीक्षा अपने पूर्व निर्धारित तिथि, समय एवं स्थान पर हीं आयोजित होंगी ।

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–IV

(Topology)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define boundary of a set. Show that a set A of a topological space (X, T), is open iff $b(A) = \overline{A} - A$.
   (b) Let A and B be sub-sets of a topological space X, then show that $b(A \cup B) \subseteq b(A) \cup b(B)$.

2. (a) Define metric topology and exhibit all conditions under metric law considered.
   (b) By introducing derived set, show that
   (i) $A \subseteq B \Rightarrow D(A) \subseteq D(B)$, (ii) $D(A \cup B) = D(A) \cup D(B)$, where A and B sub-sets of a topological space *X* and *D (A)* is the derived set of *A*.

3. (a) Give an example of a non-Hausdorff space in which every convergent sequence has unique limit (Support your example with justification).
   (b) Let (Y, U) be a sub-space of a topological space (X, T) and ACY, then A is closed in (Y, U) iff $A = F \cap Y$ where F is a closed set in (X, T). Prove this statement.

4. (a) Show by example that a continuous mapping is not necessarily an open mapping.
   (b) Prove that the open rectangles in the Euclidean plane form an open base.

5. (a) If f and g are continuous real or complex valued functions defined on a topological space X. Show that the functions $f + g$, αf *and* fg (defined pointwise), are also continous.
   (b) Show that the property of being a T_2–space, is both hereditary and topological.

6. (a) Prove that a topological space is normal iff each neghbourhood of a closed set F contains the closure of some neghbourhood of F.
   (b) Show that every metric space is a normal space.

7. (a) Prove that a one-to-one continuous mapping of compact space into a Hausdorff space, is a homeomorphism.
   (b) Show that every co-finite space is compact.

8. (a) Show that every compact regular space is normal.
   (b) Show that every compact sub-space of the real line is closed and bounded.

9. (a) Let (X, T) be a topological space and A is a connected sub-space of X. Show that $\overline{A}$, the closure of A is connected.
   (b) Let X be a topological space and $Y \subseteq X$ be a connected sub-set such that $Y \subseteq A \cup B$, where A and B are separated sets, then show that either $Y \subseteq A$ *or* $Y \subseteq B$ *i.e.* Y can't intersect both A and B.

10. (a) Any continuous image of a connected space is connected.
    (b) Let X = {a, b, c, d} and T = {ϕ, X, {b}, {b, c}, {b, c, d}} show that X is connected.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–V

(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that *if V(F)* is an n-dimensional space, then *V* is isomorphic to F^n.
   (b) Let $V_3(R)$ be a vector-space and $B\{(1,-1,3),(0,1,-1),(0,3,-2)\}$ be a basis of $V_3(R)$. Find its dual basis.
2. If $V = W_1 \oplus W_2 - - - \oplus W_k$, then show that, there exists linear operators $E_1, E_2, - - - E_k$ on *V* such that (i) Each E_i is a projection, (ii) $E_i E_j = 0$ *if* $i \neq j$, (iii) $E_1 + E_2 + - - - + E_R = I$, (iv) Range $E_i = W_i$ and conversely.
3. (a) If $B_1 = \{\alpha_1, \alpha_2 - - - -, \alpha_n\}$ be a basis of n-dimensional m-dimensional space *U(K)* and $B_2 = \{\beta_1, \beta_2, - - - -, \beta_m\}$be a basis of m-dimensional space *V(K)* and $\{a_{ij}\}$ is set of *nm* scalars $\{i = 1, 2, - - - n \text{ and } j = 1, 2, - - - m\}$, then show that there exists a unique bilinear form on $U \oplus V$ such that $f(\alpha_i, \beta_j) = a_{ij}$ for all $i = 1, 2, - - - n$ *and* $j = 1, 2, - - - m$.
   (b) Define a linear functional on a vector space. Let *R* be a field and $a_1, a_2, - - -, a_n$ be fixed scalars in *R*. Define a function *f* on R^n by $f(x) = f(x_1, x_2, - - -, x_n) = a_1x_1 + a_2x_2 + - - + a_nx_n$ then show that *f* is a linear functional on R^n.
4. (a) If *f* is a linear functional on a vector-space *V(K),* then show that (i) $f(0) = 0$ and (ii) $f(-x) = -f(x), \forall x \in V$.
   (b) Prove that two real quadratic forms are equivalent iff they have the same rank and same index.
5. (a) Define leart upper bound and greatest lower bound on a partially ordered set $(X, \subseteq)$. Prove that the partially ordered set $(P(X), \subseteq)$ is a lattice.
   (b) Let *R* be a ring and *L* be the lattice of all ideals of *R*, then show that *L* is a modular.
6. (a) The necessary and sufficient condition for a bijective map between two lattices L_1 and L_2 to be an isomorphism is that f and f^{-1} are both order preserving.
   (b) Prove that the family of all topologies on a set forms a complete lattice.
7. Define Boolean ring and Boolean algebra. Show that a Boolean Ring can be mode into a Boolean Algebra.
8. (a) Prove involution law $x = (x^1)^1$ if (i) $x \vee x^1 = 1$, (ii) $x \wedge x^1 = 0$, (iii) $x^1 \vee x'' = 1$ and $x' \wedge x'' = 0$.
   (b) Prove that the intersection of any two sub-algebras of a Boolean Algebra *B*, is a Boolean sub-algebra of *B*.
9. (a) Introduce matrix representation of a linear mapping.
   (b) prove that the similarity relation in the set of all linear operators on a vector space *V(K)* is an equivalence relation.
10. (a) If T_1 and T_2 be similar linear transformations on a finite dimensional vector space *V(K)*, then show that det T_1 = det T_2.
    (b) Reduce the matrix $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$ to Jordan canonical form.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–VI

(Complex Analysis)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Derive necessary conditions for a function to be analytic in Cartesian form.
   (b) If $w = f(z) = u + iv$ and $u - iv = e^{x}(Cos\, y - Sin\, y)$, find w as a function of z.

2. (a) Explain circle of convergence of a power series and a sophiscated method for finding it.
   (b) Examine the behaviour of the power series $\sum_{n=2}^{\infty} \frac{z^n}{n(\log n)^2}$ on its circle of convergence.

3. (a) Explain a bilinear transformation and describe its critical points in different situations.
   (b) Find all Mobius transformations which transforms unit circle $|z| \leq 1$ into unit circle $|w| \leq 1$.

4. Show that continuity is a necessary condition for differentiability but not sufficient. How a function can be enriched so as to become differentiable.

5. (a) State and prove Liouirlle's Theorem.
   (b) Evaluate $\int_C \frac{dz}{z(z + \pi i)}$, where C is the circle $|z + 3i| = 1$.

6. (a) What is Cauchy's integral formula ? Derive its establishment.
   (b) Use nth derivative integral formula for deriving Cauchy's inequalities.

7. (a) State and prove Morera's Theorem.
   (b) Prove that the function $Sin\left\{C\left(z + \frac{1}{z}\right)\right\}$ can be expanded in a series of the type $\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^{-n}$ in which the coefficients of both z^n and z^{-n} are $\frac{1}{2\pi}\int_0^{2\pi} Sin(2c\, Cos\,\theta)\, Cos\, n\theta\, d\theta$.

8. (a) Explain and establish Rouche's theorem.
   (b) Using residue theorem, evaluate $\int_C \frac{e^z\, dz}{z(z-1)^2}$, where C is the circle $|z| = 2$.

9. (a) If $a \in I(c)$, then show that $G(z, a) > 0$ for each z in $I(c)$ such that $z \neq a$.
   (b) Find the solution of Dirichlet's problem, for $t = e^{i\phi}$ on the unit circle C and $f\left(e^{i\phi}\right) = \begin{cases} 0, & if\ 0 < \phi < \pi \\ 1, & if\ \pi < \phi < 2\pi \end{cases}$.

10. What kind of singularities have the following functions :—
    (i) $\frac{1}{Sin\, z - Cos\, z}$ at $z = \pi/4$
    (ii) $Sin\, z - Cos\, z$ at $z = \infty$
    (iii) $\frac{1 - e^z}{1 + e^z}$ at $z = \infty$
    (iv) $z\, Cosec\, z$ at $z = \infty$

    Justify the answer by full logic.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–VII

(Theory of Differential Equations)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. State and prove Cauchy-Peano Existence theorem.

2. (a) Let $f \in (c, Lip)(D)$ and let $(r, s) \in D$. Then show that there exists an interval I containing r such that there exists a unique solution $g \in c[I]$ such that $g^1(x) = f(x, g(x)),\ x \in I$ and $g(r) = s$.

   (b) Compute the first three successive approximate for the solution of the equation $y' = \frac{y}{1 + y^2},\ y(0) = 1$.

3. (a) Define a linear system and show that it satisfies Lipschitz condition and set of solutions form a vector space.

   (b) Show that the following function satisfy the Lipschitz condition in the rectangle indicated and then find the Lipschitz constant.
   $f(x, y) = (y + y^2)\frac{Cos\,x}{2},\ |y| \le 1\ \&\ |x - 1| \le \frac{1}{2}$

4. (a) Let $A(x)$ be continuous function on $[a, b]$. Then, prove that the initial value problem (I.V.P.) $\frac{dy}{dx} = A(x)\,y,\ y(r) = s,\ a \le r \le b, |s| < \propto$ has unique solution on $[a, b]$.

   (b) Solve $y_1' = 2y_1 + y_2$ and $y_2' = 3y_1 + 4y_2$.

5. (a) Introduce the concept of e^A, where A is a square matrix of order n show that $|e^A| \le (n - 1) + e^{|A|}$.

   (b) Find e^A, where $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

6. (a) Define fundamental matrix and show that a necessary and sufficient condition that a solution matrix G to be a fundamental matrix, is that $let\,G(x) \ne 0$, for $X \in I$.

   (b) Solve the system of Differential Equations by matrix method. $\frac{dx_1}{dt} = 9x_1 - 8x_2$, $\frac{dx_2}{dt} = 24x_1 - 19x_2$ and initial conditions are $x_1(0) = 1,\ x_2(0) = 0$.

7. Define stable and asymptotically stable solutions. Let $x' = Ax + f(t, x)$, where A is a real constant matrix with the characteristic roots are having negative real parts. If f be a real continuous function for small $|x|$ and $t \ge 0$ as well as $f(t, x) = 0(|x|)$, where $|x| \to 0$ uniformly in $t(t \ge 0)$, then show that the identically zero solution is asymptotically stable.

8. (a) What do you mean by critical points of a system. Find the nature of critical points *(0, 0)* of the system $\frac{dx}{dt} = 2x + y,\ \frac{dy}{dt} = 3x + 4y$.

   (b) Test the stability of the non-linear system $\frac{dx}{dt} = x + 4y - x^2,\ \frac{dy}{dt} = 6x - y + 2xy$. Also comment on the type of stability.

P.T.O.

9. (a) If $P_n(x)$ represents the Legendre function, then show that,

$$(2n+1)x\,P_n(x) = (n+1)\,P_{n+1}(x) + nP_{n-1}(x).$$

(b) Derive orthogonal property of Laguerre polynomials.

10. (a) Give the series form value of a Bessel's function $J_n(x)$ and show it satisfies the Bessel's equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0$.

(b) Prove the following relations :—

(i) $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer.

(ii) $e^{\frac{x}{2}\left(z - \frac{1}{z}\right)} = \sum_{n=-\infty}^{n=\infty} z^n J_n(x)$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–I, PAPER–VIII

(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Show that the set of all real numbers, is uncountable.
   (b) Prove that every set can be well ordered.
2. (a) If A and B are two countable sets, then show that the Cartesian product $A \times B$ is also countable.
   (b) Prove that Zorn's lemma implies well ordering theorem.
3. (a) Prove the relation $V - E + R = 2$, where symbols have their usual meaning in Graph Theory and the graph G taken, is connected.
   (b) Show that a complete graph of n vertices is planner if $n \le 4$.
4. (a) Prove that an undirected graph is a tree iff there is unique path between any two vertices.
   (b) If a tree has n vertices of degree 1, two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the value of n.
5. (a) State and prove division algorithm in theory of numbers.
   (b) Solve the following system of linear congruencies.
   $x \equiv 3 \pmod{11}$
   $x \equiv 5 \pmod{19}$
   $x \equiv 10 \pmod{29}$
6. (a) State and prove Euler's criterion for an integer 'a' to be a quadratic residne modulo p (a prime).
   (b) Find (24, 63) as a linear combination of 24 and 63.
7. (a) Define osculating plane and derive the vector and scalar equations for it at a point on the space curve.
   (b) For the curve defined as $\vec{r} = \left[a\left(3u - u^3\right), 3au^2, a\left(3u + u^3\right)\right]$, prove that the curvature and torsion are equal.
8. (a) Define spherical indicatrices. Then find the curvature and torsion of the shperical indicatrix of binormals.
   (b) By defining Bertarand curves, derive important results among them.
9. (a) What do you mean by conjugate directions at a point of the surface ? Prove that at any point on the surface the sum of the radii of normal curvature in conjugate directions, is constant.
   (b) Prove that indicatrix at every points of the helicoid $z = a \tan^{-1}\left(y/x\right)$, is a rectangular hyperbola.
10. (a) Define an umbilic. Prove that in general three lines of curvature pass through an umbilic.
    (b) Show that for a geodesic, $T^2 = \left(K - K_1\right)\left(K - K_2\right)$, where K is curvature and T is torsion of the geodesic.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–IX

(Numerical Analysis)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) State and prove Newtoms Backward difference formula.
   (b) Estimate the population for the year 1905, using Newtoms formula for interpolation.

Year	1891	1901	1911	1921	1931
Population	98752	132285	168076	195690	246050

2. (a) State and prove Stirleng's formula.
   (b) Using Lagrange's interpolation formula. Find the form of the function *f(x)* from the table given below.

x	0	1	3	4
y	–12	0	12	24

3. (a) Prove that the *n*th divided difference can be expressed as the quotient of the determinants each of order $(n + 1)$.
   (b) From the given table evaluate $f(7.5)$

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

4. Find the first and second derivations of the function tabulated below at the point $x = 1.1$.

x	1	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.1280	0.5440	1.2960	2.4320	4.00

5. (a) Assuming Stirling's formula, obtain the following approximation
   $\frac{d}{dx} f(x) = \frac{2}{3}[f(x+1) - f(x-1)] - \frac{1}{12}[f(x+2) - f(x-2)]$ upto third differences.
   (b) Using divided differences, find the value of $f^1(8)$, given that $f(16) = 1.556$, $f(7) = 1.690$, $f(9) = 1.908$, $f(12) = 2.158$.
6. (a) Prove that $\nabla \Delta = \Delta - \nabla$.
   (b) Evaluate the missing term in the following :—

x	100	101	102	103	104
log x	2.000	2.0043	—	2.0128	2.0170

7. (a) The equation $x^6 - x^4 - x^3 - 1 = 0$ has one root between 1.4 and 1.5. Find the root to four decimal places by false position.
   (b) Obtain square root of 11 to five places of decimal by Newton's Raphson method.
8. (a) solve $y_{x+2} - 7y_{x+1} - 8y_x = (x^2 - x)2^x$.
   (b) Fit a second degree parabola to the following data :—

x :	0	1	2	3	4
y :	1	5	10	22	38

9. (a) State and prove Trapezoidal Rule.
   (b) Calculate an approximate value of integral $\int_0^{\pi/2} Sin\, x\, dx$.
10. (a) Calculate the values of the integral $\int_4^{5.2} \log x\, dx$ by Simpsons $\frac{1}{3}$rd rule.
    (b) Show that the difference equation $y_{x+2} - 4y_{x+1} + 4y_x = 0$, $x = 0, 1, 2, 3,$ .... has the solution $y_x = 2^x(c_1 + xc_2)$ $x = 0, 1, 2,$ .... for any constants.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics
PART–II, PAPER–X
(Functional Analysis)
Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Let N be a normal linear space and 'd' be a function on $N \times N$ to R^+ (set of non-negative real numbers) defined as $d(x, y) = \| x - y \|, \forall\, x, y \in N$. Then, show that '$d$' is a metric on N.
   (b) Consider a real number p such that $1 \le p < +\infty$. Denote l_p the space of all sequences $x = (x_1, x_2, --- x_n, ---)$ of scalars such that $\sum_{n=1}^{\infty} |x_n|^p < \infty$, with the norm defined by $\| x \|_p = \left(\sum_{n=1}^{\infty} |x_n|^p \right)^{\frac{1}{p}}$. Show that l_p is a Banach space.
2. (a) Show that a normed linear space, is a metric space, under the property $\big| \| x \| - \| y \| \big| \le \| x - y \|$.
   (b) Let p be a real number such that $1 \le p < \infty$. Denote l_∞^n the space of all n-tupples of scalars (real or complex) as $x = (x_1, x_2, ---, x_n)$ with the norm defined by $\| x \| = \max \{ |x_1|, |x_2|, ---, |x_n| \}$. Show that l_∞^n is a Banach space.
3. Define quotient space. Let M be a closed linear sub-space of a normed linear space N. If the norm of a coset $x + M$ in the quotient space N/M is defined by $\|x+M\| = Inf\{\|x+m\| : m \in M\}$. Then show that N/M is a normed linear space. Moreover, in case N also happens to be a Banach space, then prove that N/M is also a Banach space.
4. State and prove Hahn–Banach theorem.
5. (a) If X and Y are Banach spaces and if T is continuous linear transformation of X into Y. Then show that T is an open mapping.
   (b) Establish the following relations :—
   (i) $(\alpha T_1 + \beta T_2)^* = \alpha T_1^* + \beta T_2^*$ (b) $(T_1 T_2)^* = T_2^* T_1^*$, where T^* is the conjugate operator of T.
6. (a) Define the dual space of a normed linear space (or a Banach space). Prove that dual space of l_1 *is* l_x.
   (b) State and prove Riesz's lemma.
7. (a) Introduce the concept of an inner product space. Prove that every inner product space, is a normed linear space.
   (b) If x and y are two vectors of a Hilbert space H. Then, show that $\| x + y \|^2 + \| x - y \|^2 = 2 \left(\| x \|^2 + \| y \|^2 \right)$.
8. (a) Show that the parallelogram law is not true in $l_1^n \ (n > 1)$.
   (b) Give an example of a Banach space which is not a Hilbert space. Justify your answer by clear logic.
9. (a) Prove that an operator T on H (Hilbert space), is normal iff $\| T^*(x) \| = \| T(x) \|$, for all $x \in H$.
   (b) If T is an operator on H. Then, the following conditions are equivalent to one another : (i) $T^* T = I$, (ii) $\| T(x) \| = \| x \|$ for all $x \in H$.
10. If $\{e_1, e_2, ---, e_n\}$ is an orthonormal set in a Hilbert space H and if x is an arbitrary element in H, then $x = \sum_{i=1}^{n} (x, e_i)\, e_i$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XI

(Partial Differential Equations)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Find the general solution of linear partial differential equation $px(x+y) = qy(x+y) - (x-y)(2x+2y+z)$
   (b) Construct the general integral of the equation $(x-y)p + (y-x-z)q = z$ and the particular solution through the circle $z = 1,\ x^2 + y^2 = 1$.

2. (a) Use Jacobi's method to solve $p_1 x_1 + p_2 x_2 = p_3^2$.
   (b) Apply Charpit's method to find complete solution of $2zx - px^2 - 2qxy + pq = 0$.

3. (a) Solve the second order partial differential equation $4r - 4s + t = 16\log(x+2y)$.
   (b) Construct the general solution of $xyr + x^2 s - yp = x^3 e^y$.

4. Classify a general equation of second order partial differential equation and illustrate the method of solution in each case.

5. Reduce the following partial differential equations to canonical forms.
   (a) $\dfrac{\partial^2 z}{\partial x^2} = x^2 \dfrac{\partial^2 z}{\partial y^2}$ (b) $r + 2xs + x^2 t = 0$

6. (a) Construct the transport equation of discontinuities for a general first order quasi-linear hyperbolic system of first order equations.
   (b) A stretched string of finite length ℓ is held fixed at its ends and is subjected to an initial displacement $u(x, 0) = u_0\ Sin\dfrac{\pi x}{\ell}$. The string is released from this position with zero initial velocity. Find the resultant time dependent motion of the string.

7. (a) If v be the potential function of an attracting system at any point $P(x, y, z)$ which does not coincicle with any one of the attracting particles, then show that $\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} + \dfrac{\partial^2 v}{\partial z^2} = 0$.
   (b) Transform the Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$ in the cylindrical co-ordinates (r, ϕ, z).

8. (a) Define equipotential surface and determine the condition under which a family of surfaces be a possible family of equipotential surface in free space.
   (b) Show that the system of Co-axial Cylinders $x^2 + y^2 + 2\lambda x + c^2 = 0$ can form a system of equipotential surfaces and hence find the law of potential.

9. Find the steady state tempreture distribution in a rectangular plate of sides 'a' and 'b' insulated at the lateral surface and satisfying the boundary conditions $u(0, y) = u(a, y) = 0$ for $0 \le y \le b$ and $u(x, 0) = 0\ u(x, b) = f(x)$ for $0 \le x \le a$.

10. Construct general solution of one dimensional wave equation satisfying the given boundary and iniation conditions. Also apply the method for the case, where the ends of the string is fixed at $x = 0$ and $x = a$ and bearing the boundary conditions $y(0, t) = 0$, $y(a, t) = 0$ for all t.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XII

(Analytical Dynamics)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Derive the formula for the kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of kinetic energy.
   (b) Prove that in a simple dynamic system $T + V$ = Constant.

2. (a) Derive Lagrange's equations of impulsive motion in a holonomic dynamical system.
   (b) A bead is sliding on a uniformly rotating wire in a force free space. Find the equation of motion.

3. (a) In a dynamical system, if the time of passing from one configuration to another is prescribed, then prove that the Hamilton principle function has a stationary value along the actual path.
   (b) Derive the Lagrange's equations of motion from Hamilton's principle.

4. (a) State Hamilton's principle and construct the equation of motion of one dimensional harmonic oscillator (use Hamilton principle).
   (b) Express the principle of least action in terms of arc length of particle trajectory.

5. Explain small oscillation and describe the Lagrange's method of solution in this situation.

6. (a) Explain normal co-ordinates and describe small oscillation under constraint (in terms of normal co-ordinates).
   (b) Prove that the roots of the Lagrangian determinant in the theory of small oscillation, are real and positive.

7. (a) Define Poisson's Bracket and show that the Poisson Bracket obeys the distributive laws i.e.
   (i) $[u + v, w]_{q_r, p_r} = [u, w]_{q_r, p_r} + [v, w]_{q_r, p_r}$
   (ii) $[uv, w]_{q_r, p_r} = u[v, w]_{q_r, p_r} + [u, w]v_{\ q_r, p_r}$
   (b) State and prove Jacobi-Poisson Theorem.

8. (a) Using the invariance of Bilinear form, show that the transformation $\phi = \frac{1}{p}$ and $P = p^2 q$, is canonical.
   (b) Find the values of α and β so that the expressions $\phi = q^\alpha \, Cos \, \beta \, p$, $P = q^\alpha \, Sin \, \beta \, p$ represent a canonical form.

9. (a) Develop time and energy relation by involving Hamilton characteristic function.
   (b) Apply Hamilton Jacobi equation to solve the Kepler problem in determining the orbit and frequency of the planets moving round the sum.

10. (a) Determine the kinetic energy and moment of momentum of a rigid body rotating about a fixed axis.
    (b) Show that the motion of a particle relative to the revolving axes are given by $u = x + z\theta_2 - y\theta_3$, $v = y + x\theta_3 - z\theta_1$, $w = z + y\theta_1 - x\theta_2$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XIII

(Fluid Mechanics)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Determine the stream lines and path of the particles whose velocity components are $u = \dfrac{x}{1+t}, v = \dfrac{y}{1+t}, w = \dfrac{z}{1+t}$.

   (b) The velocity distribution of a certain two-dimensional flow is given by $u = Ay + B$ and $v = Ct$, where A, B and C are constants. Obtain the equation of path lines of fluid elements.

2. (a) A mass of fluid is in motion so that the lines of motion lie on the surface of co-axial cylinders (axis being taken as the z–axis). show that the equation of continuity is, $\dfrac{\partial \rho}{\partial t} + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}(\rho u) + \dfrac{\partial}{\partial z}(\rho v) = 0$, where u, v are the velocities perpendicular and parallel to the z–axis respectively.

   (b) Define the boundary surface of a liquid. Show that the variable ellipsoid $\dfrac{x^2}{a^2\kappa^2 t^4} + \kappa t^2\left(\dfrac{y^2}{b^2} + \dfrac{z^2}{e^2}\right) = 1$, is a possible form of boundary surface of a liquid.

3. (a) Obtain Euler's equation of fluid motion.

   (b) An elastic fluid, the weight of which is neglected, obeying Boyle's law, is in motion in a uniform straight tube. Show that on the hypothesis of parallel sections the velocity at any time t at a distance r from a fixed point in the tube is defined by the equation $\dfrac{\partial^2 v}{\partial t^2} + \dfrac{\partial}{\partial r}\left(2v\dfrac{\partial v}{\partial t} + v^2\dfrac{dv}{dt}\right) = \kappa\dfrac{\partial^2 v}{\partial r^2}$.

4. (a) Derive the equation of energy in the motion of non-viscous fluid.

   (b) A pulse travelling along a fine straight uniform tube filled with gas causes the density at time t and distance x from the origin where the velocity is u_0 to become $\rho_0\phi(vt - x)$ prove that the velocity u (at time t and distance x from the origin), is given by $v + \dfrac{(u_0 - v)\,\phi\,(vt)}{\phi(vt - x)}$.

5. (a) Show that in two dimensional irrotational motion, stream function satisfies Laplace's equation.

   (b) Derive Canchy-Riemann differential equation in polar form.

6. (a) A two dimensional flow field, is given by $\psi = xy$.
   (i) Show that flow is irrotational.
   (ii) Find the velocity potential.
   (iii) Prove that ψ and ϕ satisfy the Laplace equation (ϕ is the velocity potential).
   (iv) Find the stream lines and potential lines.

   (b) The velocity potentials $\phi_1 = x^2 - y^2$ and $\phi_2 = \sqrt{r}\,Cos\dfrac{\theta}{2}$ are solutions of Laplace equation. Prove that $\phi_3 = x^2 - y^2 + \sqrt{r}\,Cos\dfrac{\theta}{2}$ satisfies $\nabla^2\phi_3 = 0$.

7. (a) Describe the general motion of a cylinder of any cross section.

(b) Discuss the motion in the case of a liquid streaming past a fixed circular cylinder of radius a with velocity U.

8. (a) Show that when a sphere of radius a with uniform velocity U through a perfect incompressible infinite fluid, the acceleration of a particle of the fluid at *(r, 0)*, is $3U^2\left(\frac{a^3}{r^4}-\frac{a^6}{r^7}\right)$.

(b) A sphere of radius 'a' is made to move in incompressible liquid (perfect) with non-uniform velocity U along the x–axis. If the pressure at infinity is zero, prove that at a point x in advance of centre, the pressure $p=\frac{1}{2}\rho a^3\left[\frac{U}{x^2}+U^2\left(\frac{2}{x^3}-\frac{a^3}{x^3}\right)\right]$.

9. (a) Define stress and strain tensors. Find the relation between stress tensor and rate of strain tensor.

(b) Show that the velocity field defined at a point ρ by $1+2y-3z$, $4-2x+5z$, $6+3x-5y$, represents a rigid body rotation.

10. Derive Navier–Stokes' equation of motion of viscous fluid and find solution of Navier-Stokes equation for incompressible fluid.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XIV

(Operations Research)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) A set in R^2 is defined as $S = \{(x_1, x_2) : x_1 x_2 \geq 1, x_1 \geq 0, x_2 \geq 0\}$. Prove that S is a convex set.
   (b) An animal fodder company needs to produce 500 kg of a mixture having components A and B cost Rs. 5 and Rs. 6 per kg respectively. The ingrient A should not exceed 90 kg and B must not be below 50 kg. Formulate this problem as a L.P.P. and find the minimum cost mixture (by using graphical method).

2. (a) Define degenerate basic and non-degenerate basic solution of a system. Form all basic feasible solution of $x_1 + 2x_2 + 4x_3 + x_4 = 7$ and $2x_1 - x_2 + 3x_3 + 2x_4 = 4$ and testify for degenerality.
   (b) Solve graphically the L.P.P.
   $Min\ z = 5x_1 + 3x_2$
   Subject to : $x_1 + x_2 \leq 6, 2x_1 + 3x_2 \geq 6, 0 \leq x_1 \leq 4$ *and* $0 \leq x_2 \leq 3$.

3. Use simplex method to solve the L.P.P.
   $Max\ z = 4x_1 + 10x_2$
   Subject to : $2x_1 + x_2 \leq 50, 2x_1 + 5x_2 \leq 10, 2x_1 + 3x_2 \leq 90; x_1 \geq 0, x_2 \geq 0$.

4. Apply two-phase method to compute the solution of $Min\ z = \frac{15}{2} x_1 - 3x_2$
   Subject to : $3x_1 - x_2 - x_3 \geq 3, x_1 - x_2 + x_3 \geq 2$ *and* $x_i \geq 0\ (i = 1, 2, 3)$.

5. (a) Prove that the dual of the dual of a primal problem is the primal problem it self.
   (b) Obtain the dual problem of the L.P.P.
   $Min\ z = 2x_1 + 3x_2 + x_3$
   Subject to : $2x_1 + 3x_2 + 5x_3 \geq 2, 3x_1 + x_2 + 7x_3 = 3, x_1 + 4x_2 + 6x_3 \leq 5$.

6. Describe in brief the dual simplex method and apply it to solve the L.P.P. given as under.
   $Min\ z = 3x_1 + x_2$
   Subject to : $x_1 + x_2 \geq 1, 2x_1 + 3x_2 \geq 2$ *and* $x_1 \geq 0, x_2 \geq 0$.

7. Examine the sensitivity of the optimal solution of a L.P.P. with regard to change in an element of coefficient matrix.

8. (a) Describe the method of constructing the solution of 'Game' problem, where the game is without saddle point.
   (b) Solve the game problem whose pay off matrix, is $\begin{bmatrix} 6 & 2 & 7 \\ 1 & 9 & 3 \end{bmatrix}$.

9. (a) Describe the solution of quadratic programming problem with linear constraints.
   (b) Use Fibonacci method to solve non-linear programming problem :—
   $Min\ z = f(x) = 0.65 - \frac{0.75}{1 + x^2} - 0.65\ x \tan^{-1} \frac{1}{x}$ in interval $[0, 3]$ using $n = 6$ and $\in = 0.01$.

10. Explain Fibonacci method of solution for non-linear programming problem.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XV

(Tensor Algebra, Integral Transform, Linear Integral Equations, Operation Research Modeling)

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Prove that the inner product of the tensor (A_s^p) and (B_t^{rs}) is a tensor of rank three.
   (b) Define reciprocal of a symmetric tensor of order two. Show that the reciprocal of a symmetric covariant tensor (a_{ij}), is a symmetric contra variant tensor of order 2.
2. (a) What do you mean by Christoffel symbols ? Prove that $[ij, k] + [jk, i] = \frac{\partial}{\partial x^j} g_{ik}$.
   (b) Show that the covariant derivative of a contra variant vector, is a mixed tensor of rank two.
3. (a) Find the Laplace Transform of the function *F(t) = Cosh at.*
   (b) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{1}{s^2(s^2-a^2)}\right\}$.
4. (a) Find the relation between Fourier transform and Laplace transform.
   (b) Express the function $f(x) = \begin{cases} 1, |x| \le 1 \\ 0, |x| > 0 \end{cases}$ as a Fourier Integral.
5. (a) Discuss Fredholm integral equation and Valterra integral equation.
   (b) Verify that the function $u(x)=1-x$, is a solution of the integral equation $\int_0^x e^{x-1} u(t)\, dt = x$.
6. (a) Form a Voltera integral equation corresponding to the differential equation given by $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 3y = 0$ with initial conditions $y(0) = 1$ and $y'(0) = 0$.
   (b) Define orthogonality of two functions on an interval [a, b]. If *k(x, t)* is symmetric $f_1(x)$ and $f_2(x)$ are eigenfunctions of *k(x, t)* corresponding to eigenvalues λ_1 and λ_2 $(\lambda_1 \ne \lambda_2)$ respectively. Then show that the functions $f_1(x)$ and $f_2(x)$ are orthogonal on [a, b].
7. The maintenance cost and resale value per year of a machine whose purchase price is Rs. 7000/– is given below :–

Year	1	2	3	4	5	6	7	8
Maintenance cost in Rs.	900	1200	1600	2100	2800	3700	4700	5900
Resale value in Rs.	4000	2000	1200	600	50	400	400	400

   When should the machine be replaced ?

8. A readymade garment manufacturer has to process 7 items through two stages of production viz cutting and sewing. The time taken for each of these items at different stages is given below in appropriate units :–

Item	1	2	3	4	5	6	7
Cutting time	5	7	3	4	6	7	12
Sewing time	2	6	7	5	9	5	8

   Find an order in which these items to be processed through these stages so as to minimize total processing time.

9. Describe the Deterministic Model with Instantaneous Production (shortage not allowed).
10. Discuss Poisson's Queing Systems Model 2 (M/M/I) : (N/FIFO) System.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics

PART–II, PAPER–XVI

(Programming in 'C')

Annual Examination, 2012

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. What is an Operator? Describe different types of operators that are included in C with examples.

2. Are the library functions actually the part of the C language? Explain. How are library functions accessed.

3. Explain the difference between **while** loop and **do-while** loop in C with examples. What is the purpose of switch statement in C? Explain with the help of an example.

4. Write a program to multiply 3x3 matrices in C.

5. How string variables declared and initialized? Explain giving a proper example.

6. What is a function? State three advantages of using functions. What is the purpose of return statement.

7. What is recursion? Write a program to find the roots of a quadratic equation.

8. What is the purpose of **typedef** feature. Explain with the help of an example how this feature is used in **structure**.

9. Write a program that swaps two variables using pointers. How is the variable's address determined.

10. Write short notes on any *Three* of the following :—
    (i) Micro Substitution
    (ii) Conditional operator
    (iii) Branching statements
    (iv) Pointer to function.

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M.Sc. Mathematics, Part–II *Practical Counselling & Examination Programme of Paper–XVI (20 Marks)*		
—: Practical Counselling Programme :—		
Date : 18.06.2012 to 22.06.2012	&	Time : 10.30 AM to 12.30 PM
—: Practical Examination Programme :—		
Date : 23.06.2012	&	Time : 12.00 Noon to 3.00 PM