Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-I
Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *Five* Questions, selecting at least one question from each group.

Group-A

1. (a) State and prove fundamental theorem on equivalence relation.
   (b) If R is an equivalence relation on a set A, then prove that R^{-1} is also an equivalence on A.
2. (a) Let $f:A \to B$ and $g:B \to C$ be one-one onto mappings then show that $g o f:A \to C$ is also one-one onto and $(g o f)^{-1} = f^{-1} o g^{-1}$.
   (b) Prove that the set of all real numbers is uncountable.

Group-B

3. (a) Define inverse of a matrix. Show that the necessary and sufficient condition for the existence of the inverse of a square matrix, is that it must be non-singular.
   (b) Find the adjoint and inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

4. (a) Find the rank of a the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$
   (b) Show that the equations x+2y+3z=14, 3x+y+2z = 11, 2x+3y+z=11, are consistent and then solve them.

Group-C

5. (a) Define an abelian group. Give two examples of an abelian group.
   (b) Show that the set of all cube roots of unity under multiplication forms an abelian group.
6. (a) Show that every group of prime order is cyclic.
   (b) Prove that the intersection of any two normal sub-groups of a group G, is a normal sub-group of G.
7. State and prove second isomorphism theorem on a group.
8. Prove that the set { $a + b\sqrt{p}$, where a and b are real numbers and p is a prime} forms a commutative ring under usual addition and multiplication. Also this forms a field with respect to the same binary operations.

Group-D

9. (a) Find the condition that the roots of the equation $x^3-px^2+qx-r=0$ be in H.P. Show that the mean (harmonic) root is $\frac{3r}{q}$. Hence, solve the equation $6x^3-11x^2-3x+2=0$.
   (b) If $\alpha, \beta, \gamma, \partial$ be the roots of the be quadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, then compute $\sum \alpha^2 \beta^2$.
10. Solve general solution of cubic equation by cardon's method.

Group-E

11. (a) Prove that the nth roots of unity forms a geometric series.
    (b) Expand Cosα in the ascending powers of α.
12. (a) If $\sin x = n \sin(x+\alpha), n < 1$. Expand x in a series of ascending powers of n.
    (b) Find the sum of the series

$$\tan^{-1}\frac{1}{2.1^2}+\tan^{-1}\frac{1}{2.2^2}+\tan^{-1}\frac{1}{2.3^2}+\ldots\ldots\ldots\ldots\ldots \text{ to n terms}$$

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Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-I
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *Five* Questions, selecting at least one question from each group.

Group-A

1. (a) State and prove Leibnitz's theorem.
   (b) Find the Lagrange's form of remainder after nth tern in the expansion of e^{ax} Cos bx as the ascending powers of x.

2. (a) Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$
   (b) State and prove Euler's theorem on homogeneous function of three variables.

3. (a) Construct the polar formule for the radius curvature.
   (b) Find the asymptote of curve r = a tan θ.

Group-B

4. Evaluate any two of the following integrals:
   (i) $\int \frac{dx}{x(x^2+1)^3}$ (ii) $\int \frac{x^2 dx}{(x+1)(x+2)^2}$ (iii) $\int \frac{e^x}{e^x - 3e^{-x} + 2}$

5. (a) Establish reduction formula for $\int \tan^n x\, dx$
   (b) Show that $\int_0^1 \frac{\log(1-x)}{x} dx = -\frac{\pi^2}{6}$

6. (a) Find the perimeter of the loop of the curve $3ay^2 = x^2 (a - x)$.
   (b) Compute the volume of the solid obtained by revolving the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its major axis.

7. Determine the surface area formed by the revolution of the cycloid $x = a(\theta + \sin\theta),\ y = a(1 - Cos\theta)$ round the tangent at the vertex.

Group-C

8. (a) Find the condition under which a general equation of second degree represents an ellipse.
   (b) An ellipse of semi-axes a and b touches the x-axis at origin. Compute the locus of its centre.

9. (a) Construct the polar equation of a conic having length of its rectum $2l$ eccentricity e and focus is the pole of the Co–ordinate system.
   (b) A circle passes through the focus of a conic of latus rectum 10 meters, meets it in four points whose distances from the focus are r_1, r_2, r_3 and r_4. Prove that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{5}$.

Group-D

10. (a) Find the equation of a sphere passing through four given points.
    (b) Obtain the equation of the Enveloping cone to surface (chosen as per your convenience) and through a point (acting as the vertex of the cone).

11. (a) Derive the equation of the cylinder generated by lines parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$, the guiding curve being the conic $ax^2+by^2=1, \quad z=0$.

(b) Find the equation of planes passing through the intersection of planes $x+y+z=2$ *and* $2x+y-z=7$ as well as touching the ellipsoid $7x^2+5y^2+3z^2=60$.

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Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Subsidiary), Part-I
Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *Five* Questions, selecting at least one question from each group.

Group-A

1. (a) Define indexed family of sets. If dash (') denotes the complement of set, then show that $\left(\bigcup_{i\in I} E_i\right)' = \left(\bigcap_{i\in i} E_i\right)$
   (b) Define a reflexive, asymmetric relations. Construct a symmetric relation which is not reflexive.

2. (a) What do you mean by an abelian group? Produce an example of a group which is not abelian.
   (b) Define a cyclic group? Show that every cylic group, is necessarily abelian.

3. (a) State and prove De–Moivre's theorem for an index.
   (b) Reduce $(\alpha + i\beta)^{x+iy}$ in the A + iB form.

4. (a) Expand $\sin\alpha$ in the ascending powers of α.
   (b) Prove that Log(–1) = $\left(\frac{4n+3}{2}\right)\pi i,$ when $n \in Z.$

Group-B

5. (a) Show that the radical axis of two circles is pendicular to the line joining their Centres.
   (b) Find the condition for the tangency of a line y = mx + c to the parabola $y^2 = 4ax$.

6. Find the conditions for the general equation of second degree representing a Conic.

7. (a) Prove that the infinite series, $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots\ldots\ldots\ldots \ to\, \alpha$ is Convergent if p > 1 and divergent when $p \le 1$.
   (b) Test the convergence of the series $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \ldots\ldots\ldots\ldots\ldots \ to\ \alpha\, (x > 0).$

Group-C

8. (a) If $y = \sin\left(m\sin^{-1} x\right)$, then show that $\left(1 - x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0.$
   (b) Find the nth derivative of $e^x \sin x \sin 2x$.

9. (a) State and prove Taylor's Theorem (Series).
   (b) If u be a homogeneous function of two variables x and y of degree n, then prove that
   $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

10. (a) Give geometrical meaning of scalar product of three vectors.
    (b) Shat that $\left(\vec{b}\ X\ \vec{c}\ \right) X \left(\vec{c}\ X\ \vec{a}\right) = \left[\vec{a}, \vec{b}, \vec{c}\right]\vec{c}$

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Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-II
Paper-III

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *Six* Questions, selecting at least one question from each group.

Group-A

1. (a) Define sum of two Dedikind Cuts. Show that the sum of two such cuts is also a Dedikind Cut.
   (b) Prove that any non-empty set of real numbers which bounded above has a least upper bound.
2. (a) State and prove Heine-Borel Theorem.
   (b) Show that the real line R is not a compact space.
3. (a) Show that function which is continuous on a closed and bounded interval is also uniformly continuous on that interval.
   (b) Prove that a function f: R $\longrightarrow$ R is continuous on R iff for every open set G in R, the set f^{-1} (G) is open in R.
4. State and prove Taylor's Theorem and state remainder in the Taylor's series. Explain different kinds of remainder.
5. (a) Explain improper integral and its convergence.
   (b) Prove that $\Gamma(n)\Gamma(1-n)=\dfrac{\pi}{\sin n\pi}(0<n<1)$.

Group-B

6. (a) Define convergence of a sequence. Show that the sequence (a_n), where $a_2=\left(1+\dfrac{1}{n}\right)^n$ is convergent.
   (b) State and prove Caunchy's General Principle of Convergence.
7. (a) State and prove Raabe's Test of Convergence.
   (b) Examine the Convergence of the series $\sum_{n=1}^{\alpha}\dfrac{(n+1)(n+2)}{(n+3)\ (n+4)}$
8. (a) Explain the De Morgan and Bertrand test and then prove it.
   (b) Satisfy yourself that the series: $\dfrac{1^2}{2^2}+\dfrac{1^2.\,3^2}{2^2.4^2}x+\dfrac{1^2.\,3^2.5^2}{2^2.4^2.6^2}x^2+.............to\ x$ where x > 0, is Convergent when X < 1 and divergent when $x\geq 1$.
9. (a) If $\sum y_n$ Converges and (x_n) is Convergent monotonic sequess, then show that $\sum x_n\ y_{n,}$ is Convergent.
   (b) Prove that the series $\sum_{1}^{\alpha}\dfrac{(-1)^{n-1}}{n}$ is conditionally Convergent. Rearrange terms of this series so that it Converges to $\dfrac{1}{2}$log6.

Group-C

10. (a) If V is a vector space over the field F, then show that,
    (i) a0 = 0, $a\in F$ and 0 is the zero vector V.
    (ii) a(-x) = -(ax), $a\in F\ \&\ x\in V$.
    (iii) a(x-y) = ax - ay, $a\in F\ and\ x, y\in V$.
    (b) A necessary and sufficient condition for a non-empty sub-set W of a vector space V over the field F, is that for all x, y $\in W\ and\ a, b\in F\ \Rightarrow ax+by\in W$. Prove this statement.
11. (a) Define a basis of a vector space. Prove that the set {(1,2,1), (2,1,0), (1,-1,2)} forms a basis of V_3 (R).
    (b) Make out difference between linear transformation and linear operator. Support your answer by suitable example.

12. (a) Prove that the eigen values of a hermitian matrix are all real.

(b) Find the characteristic equation of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and verify Cayley-Hamiltion Theorem. Hence, calculate the inverse of A.

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Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-II
Paper-IV

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *Six* Questions, selecting at least one question from each group.

Group-A

1. (a) Solve any two of the following:
(i) $p^2+2ypCotx=y^2$ (ii) $y=2px+y^2p^3$ (iii) $y=px+p-p^2$
(b) Find the orthogonal trajactory of the family of cardoids $r = a(1+Cos\theta)$.

2. (a) Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = x^3$ (b) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$

3. Solve any two of the following:
(i) $x\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$ (ii) $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2-3)y = e^{x^2}$
(iii) $\frac{d^2y}{dx^2} - Cotx\frac{dy}{dx} + 4yCo\sec^2 x = 0$

Group-B

4. (a) Prove that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$
(b) Show that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

5. (a) If $\vec{r} = \vec{a}\,Coswt + \vec{b}\,\sin wt$, then prove that (i) $\vec{r}\times\frac{d\vec{r}}{dt} = w\vec{a}\times\vec{b}$ and (ii) $\frac{d^2\vec{r}}{dt^2} = -w^2\vec{r}$.
(b) Give the geometrical meaning of gradient of a scalar function.

6. (a) State and prove Green's Theorem.
(b) Show that $\iint (a\,x\,\hat{i} + b\,y\,\hat{j} + c\,z\,\hat{k}).\hat{n}\,d\,s = \frac{4}{3}\pi(a+b+c), where,\ s \equiv x^2+y^2+z^2-1=0$

Group-C

7. (a) A uniform beam of length 2a rests in equilibrium against a smooth vertical wall and with a point of its length resting against a smooth horizontal rod which perpendicular to the wall and at a distance b from it, show that the inclination of the beam with the vertical is $\sin^{-1}\left(b/a\right)^{1/3}$.
(b) Derive the general conditions of equilibrium of a Coplanar system of forces.

8. (a) State and prove principle of virtual work.
(b) Four uniform rods are freely jointed at their extremities and form a parallelogram ABCD which is suspended by the point A and is kept in shape by a string AC. Prove that the tension of the string is equal to half the whole weight.

9. (a) Describe the condition of stability for a body with one degree of freedom.
(b) A force P acts along the axis of x and another force np along a generator of the cylinder $x^2+y^2=a^2$. Show that the central axis lies on the cylinder $n^2(nx-z)^2+(1+n^2)^2y^2=n^4a^2$.

Group-D

10. (a) Define S.H.M. Write its equation of motion and describe it fully.
(b) A particle whose mass is m is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. Find the time consumed in reaching the origin.

11. (a) Deduce the formula for tangential and normal accleration of a particle in a plane.
(b) The velocity of a particle along and perpendicular to the radius from a fixed origin are λr and $\mu\theta$ respectively. Find the path and accelerations along and perpendicular to the radius vector.

12. (a) A particle moves along the curve $y = a \log \sec \frac{x}{a}$ in such a way that the tangent to the curve rotates uniformly, prove that the resultant acceleration of the particle varies as the square of the radius of curvature.

(b) Describe the motion of a particle under central forces.

Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Subsidiary), Part-II
Paper-II

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *eight* Questions, selecting at least one question from each group.

Group-A

1. (a) Evaluate any two of the following:

(i) $\int \frac{dx}{\sin x(3+2Cosx)}$ (ii) $\int \frac{dx}{\sqrt{x}+\sqrt{1+x}}$ (iii) $\int_0^{\pi/2} \frac{dx}{a+b\,Cosx}$ (a > b > 0).

2. (a) Evaluate $\int_0^1 Cosx\,dx$ as the limit of sum.

(b) Integrate $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$

3. (a) Obtain the reduction formula for $\int \tan^n x\,dx$

(b) Find $\underset{n\to\alpha}{Lim}\sum_{r=1}^{n}\frac{r^3}{r^4+n^4}$

4. (a) Calculate the length of the arc of the curve y = log x intercepted between the ordinates x = 1 and x = 2.

(b) Determine the area of the loop of the curve $x^3 + y^3 - 3axy = 0$.

5. Solve any two of the following:

(a) $y = x (p + p^2)$ (b) $y^2 \log y = xyp + p^2$ (c) Sin y Cos px - Cos y Sin px - p = 0.

6. Find the orthogonal trajectories of the family $y^2 = 4ax$

7. Find the general solution of the following equations:

(a) $(D^2 – 2D + 1) y = x^2e^{3x}$ (b) $(D^2 + a^2) y = Cos\ ax$.

Group-B

8. (a) Find the equation of the sphere whose centre is (α,β,γ) and radius r.

(b) Form the equation of the cone whose vertex is (0,0,3) and guiding curve $x^2-y^2=4,\ Z=0.$

9. Formulate the equation of the right circular cylinder whose axis is $\frac{x}{1}=\frac{y}{0}=\frac{Z}{2}$ *and radius* $\sqrt{7}.$

Group-C

10. (a) Define a convex set and prove that the sphere is a convex set.

(b) Solve the following problem graphically:

Maximize $Z=5x_1+3x_2$, subject to $3x_1 + 5x_2 \le 15$, $5x_1 + 2x_2 \le 10$ and $x_1, x_2 \ge 0$.

11. Use simplex method to solve the following L.P.P

Max $Z=7x_1+5x_2,$ Subject to : $x_1 + 2x_2 \le 6$, $4x_1 + 3x_2 \le 12$ *and* $x_1 \ge 0,\ x_2 \ge 0.$

Group-D

12. (a) Show that in a Simple Harmonic Motion $f^2T^2 + 4\pi^2v^2$ is constant, where f is acceleration, T the periodic time and v the velocity at a current point.

(b) A particle whose mass is m, is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin, where μ and a are constants and x is the distance of the particle from the origin. Find the time taken by the particle to arrive at the origin.

13. A short of mass m penetrates a thickness t of a fixed plate of mass m. Prove that, if M is free to move, the thickness penetrated is $\frac{Mt}{M+m}$.

14. Obtain the radial and transverse accelerations of a particle moving on a plane curve.

15. (a) Find the equation of the line of action of the resultant force of a system of Co-planar forces acting on a rigid body.

(b) If the algebraic sum of moments of a system of coplanar forces acting on a rigid body vanishes about each of three non-collinear points. Prove that the system is in equilibrium.

16. State and prove the Principle of virtual work.

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Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-III
Paper-V

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *Six* Questions, selecting at least one question from each group.

Group-A

1. (a) Let X be a metric space, then show that for each pair of distinct points of X, there exist neghbourhoods N_1 and N_2 such that $N_1 \cap N_2 = \phi$.

   (b) Let (X, d) be a metric space and p be defined on X x X as $\rho(x,y)=\frac{d(x,y)}{1+d(x,y)}$, then show that ρ is a metric on X.

2. (a) Let (X, d) be a metric space and $A \subseteq X$ *with* $p \in X$. Then p is an accumulation point of A iff each neghbourhood of p contains a point of A other than p. Prove this statement.

   (b) Define a derived set of a non-empty set A. Let (X,d) be a metric space and $A \subseteq X$, then prove that A' (derived set of A) is closed.

3. (a) In a metric space, show that the limit of a convergent sequence, is unique.

   (b) If x and y are two points in the metric space (X, d) and if (x_n) and (y_n) are convergent sequences in X having limit of convergence x and y respectively, then show that d (x_n, y_n) converges to d(x, y).

4. State and prove Cantor's intersection theorem.

5. (a) Let (X, d_1) and (Y, d_2) be two metric spaces and f is a mapping from X to Y. Then f is continuous iff inverse image of every open sub-set E of Y is an open sub-set of X.

Group-B

6. (a) If f be a bounded functions in [a, b] and P, P_1 are partitions of [a, b] such that P is finer then P_1. Then show that $L(P_1) \le L(P) \le U(P) \le U(P_1)$.

   (b) Prove that every monotonic functions on [a, b], is R- integrable on [a, b]

7. (a) State and prove Bonnet's form of Mean Value Theorem.

   (b) Prove that if f is continuous on [a, b], then there exists a number λ lying between a and b such that $\int_a^b f(x)dx = (b-a) f(\lambda)$.

Group-C

8. (a) State and establish Cauchy's integral test.

   (b) Apply Cauchy's integral test to test the convergence of the series $\sum_{n=2}^{\alpha} \frac{1}{n(\log n)^p}$.

9. (a) Introduce Infinite product and its convergence.

   (b) Examine the nature of infinite product Πa_n, where $a_n = \frac{n\ (n+2)}{(n+1)^2}$.

Group-D

10. (a) Let N be a normal linear space. Show that N is a Banach space iff $\{x : \| x \| = 1\}$ is complete.

    (b) Consider l^p as set of Convergent sequences of real numbers. Show that (l^p, d) is a Banach space, where d is defined by d (x, y) = $\left(\sum_{i=1}^{\alpha} |x_i - y_i|^p \right)^{1/p} = \| x - y \|$ and $\sum_{i=1}^{\alpha} | x_i |^p < \alpha, \quad \sum_{i=1}^{\alpha} | y_i |^p < \alpha$

11. (a) Let N and N^1 be two normed linear spaces. Show that the set of all continuous linear transformations of N to N^1, is a normed linear space with respect to the pointwise linear operations and norm is defined by $\| T \| = \inf \{\kappa : k \ge 0\}$ *and* $\|T(x)\| \le k \|x\|$ *for all* $x \in N$.

12. (a) Let the inner product on C $[-1, 1]$ be defined as $(f,g)=\int_{-1}^{1} f(t)g(t)dt \ for\,all \ f,\ g\in C[-1,1]$. Show that C[-1, 1] is an inner product space which is not a Hilbert space.

(b) Show that an inner product space X is an uniformly Convex.

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Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-III
Paper-VI

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *Six* Questions, selecting at least one question from each group.

Group - A

1. (a) Define centre of a group. Show that the centre Z of a group G, is a normal sub-group of G.
   (b) Let f be a function on the group G defined by $f(x) = x^{-1}$ for each x in G. Show that f is an automorphism iff G is abelian.

2. (a) Let $f: R \to T$ be an isomorphism of a ring R to a ring T. If 0 and $0'$ are zero elements of R and T respectively, then show that $f(0) = 0'$ and f(-x) = -f(x) for all x in R.
   (b) Show by an example that if I and J are ideals in R, then I U J is not an ideal in R.

3. Let I be an ideal in a ring R. Show that the quotient ring R/I is a ring. Further if R is commutative, then so is R/I.

4. Prove that every integral domain can be embedded in to a field.

Group - B

5. For any Cardinals α, β, γ show that
   (i) $\alpha^{\beta} . \alpha^{\gamma} = \alpha^{\beta+\gamma}$ (ii) $(\alpha\beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$

6. (a) If E is any set, then show that card P(E) = 2 Card E, where P(E) denotes the power set of E.
   (b) Find out the order type of the set $Q \cap (a, b)$ taken with usual order.

7. (a) State and prove Zorn's lemma.
   (b) Prove that $n + \alpha = \alpha$, where α is an infinite Cardinal number and n is a natural number.

Group - C

8. (a) Derive the formula for the Mean and Variance of a Binomial distribution.
   (b) Find the number of solutions of the equation x + y + Z = 15, where x, y, z being non-negative integers.

9. (a) Find all solutions of the recurrence relation $a_n = 3a_{n-1} + 2a_{n-2}$.
   (b) Illustrate how generating function can be used to get the number of r-Combinations from a set with n elements when repetition of elements is allowed.

Group-D

10. (a) Show that $f(Z) = \sqrt{xy}$ is not analytic at the origin although cauchy - Riemann differential equations are satisfied at the point.
    (b) Construct the analytic function f(Z) = u + i*v*, where $u = x^3 - 3xy^2 + 3x + 1$.

11. State and prove Laurrent's theorem.

12. (a) Explain singularities and its types with suitable examples.
    (b) Find the kind of singularities of the function $f(z) = \frac{Cot \pi z}{(z-a)^2}$ at z = a and z = α.

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Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-III
Paper-VII

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any *Six* Questions, selecting at least one question from each group.

Group - A

1. (a) Define convex combination of vectors in R^n. Show that the set of all Convex Combinations of a finite number of linearly independent vectors v_1, v_2, ..., v_n, is a convex set.
   (b) A tailor has 95 square meters cotton material and 145 square meters of wool material. A suit requires 1 square meter of cotton and 3 square meters of wool and dress requires 2 square meters of each. How many of each garment should the tailor produces so as to maximize his income if a suit sells for Rs 350 and a dress for Rs 145?

2. (a) Introduce degenerate and non-degenerate basic solution. Obtain all basic solution of the system.
   $x_1 + 2x_2 + x_3 = 4$, $2x_1 + x_2 + 5x_3 = 5$, by specifying the nature of degeneracy/non-degeneracy.
   (b) Use simplex method to solve the L.P.P.
   Max $Z = 3x_1 + 2x_2$
   Subject to the constraints $x_1 + x_2 \leq 4,\ x_1 - x_2 \leq 2\ and\ x_1 \geq 0,\ x_2 \geq 0.$

3. From the initial basic feasible solution of the transportation problem by using Vogel approximation method.

	W_1	W_2	W_3	W_4	Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement :	5	8	7	14	

Group - B

4. Test for integrability and hence solve the equation
$$(y^2 + yz)dx + (z^2 + xz)dy + (y^2 - xy)dz = 0.$$

5. (a) Interprate the equation Pdx + Qdy + Rdz = 0.
   (b) Solve the simultaneous equation
$$\frac{dx}{dt} + 2x - 3y = t,\ \ \frac{dy}{dx} - 3x + 2y = e^{2t}$$

6. (a) Apply Charpit's method to find the complete integral of px + qy = pq.
   (b) Solve : $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0.$

7. (a) Solve $\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}.$

(b) Use Monge's method to solve the equation.

$$r - t\cos^2 x + p\tan x = 0$$

P.T.O

Group - C

8. (a) Find the attraction of circular rod at its centre.
   (b) Prove that the attraction of a solid hemisphere at the centre of its plane base is $\frac{3}{2} X \frac{M}{a^2}$, where M is the mass and a is the radius.

9. Define equipotential surface and lines of force. Derive the condition for a family of surfaces given by $\phi(x, y, z) = c$ be a possible family of equipotential surfaces in free surface.

Group-D

10. (a) Derive the expression for pressure at a point in a heavy homogeneous fluid at rest under gravity.
    (b) A square ABCD is immersed in water with the side AB in the surface. Draw a straight line through A which shall divide the lamina into two parts, the thrust on which are equal.

11. (a) By integration method, find the centre of pressure of a lamina.
    (b) Prove that the depth below the surface of a liquid of the centre of pressure of a rectangle, two of whose parallel sides are horizontal and at dept a and b, is $\frac{2(a^2+ab+b^2)}{3(a+b)}$.

12. (a) A frustrum of a right circular cone cut off by a plane bisecting the aims perpendicularly, floats with its smaller end in water and its axis just half immersed. Prove that the specific gravity of the cone is $\frac{19}{56}$.
    (b) Define equipressure and equidensity surfaces. Derive the equations of curves of equal pressure and density.

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MOST IMPORTNAT NOTICE

: अत्यन्त महत्वपूर्ण सूचना :

बी0एस0सी0(प्रतिष्ठा)पार्ट-III एवं मनोविज्ञान(प्रतिष्ठा)पार्ट- III के सभी विद्यार्थियों को सूचित करना है कि उनके सामान्य एवं पर्यावरणीय अध्ययन **(G.S)** की परीक्षा 2012 जो पूर्व से दिनांक-23.02.2012 को द्वितीय पाली समय 12 से 3 बजे, अपराह्न में निर्धारित थी, **अब यह परीक्षा दिनांक 24.02.2012 को द्वितीय पाली समय 12 से 3 बजे, अपराह्न में संचालित होगी** । तदनुरूप परीक्षा में उपस्थित होवें।

Nalanda Open University
Annual Examination - 2012
B.Sc. Mathematics (Honours), Part-III
Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80/75**

Answer any five questions. All questions carry equal marks.
Calculator is allowed.

Group - A

1. (a) Evaluate $\frac{\Delta}{E}\sin(x+h)+\frac{\Delta^2\sin(x+h)}{E\sin(x+h)}$

   (b) By means of divided differences formula, find the values of f(2), f(8) and f(15) from the following table.

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

2. (a) Establish Newton's backward interpolation formula for equal interval length.
   (b) Find the missing term in the following table.

x	16	18	20	22	24	26
f(x)	39	15	....	151	264	388

3. (a) Find the first and second derivatives of the function y = f(x) tabulated below at the point x = 7.50.

x	7.47	7.48	7.49	7.50	7.51	7.52	7.53
y = f(x)	0.193	0.195	0.198	0.201	0.203	0.206	0.208

   (b) Define factorial notation in the Difference Calculus. Express f(x) = $2x^3 - 3x^2 + 3x - 10$ in the factorial notation the interval of differencing being unity.

4. (a) Construct general quadrature formula for equidistant values of x.

   (b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's one third rule and hence find the approximate value of π.

5. (a) Solve $2u_{x+2} - 5u_{x+1} + 2u_x = 0$ and find the particular solution when $u_0 = -1$, $u_1 = 1$.

   (b) Solve $\frac{dy}{dx}=x+\left|\sqrt{y}\right|$ by Euler's modified method with inial condition y = 1 at x = 0 for all the range $0\le x\le 0.6$ in the steps of 0.2.

6. Use Pieard's method of successive approximation for solving first order simultaneous equations.

$$\frac{dy}{dx}=x+z \text{ and } \frac{dz}{dx}=x-y^2 \text{ satisfying } y=2, z=1 \text{ at } x=0.$$

7. Solve $\frac{dy}{dx}=x+y$ with initial condition y(0) = 1 by Runge-Kutta method form x = 0 to x = 0.4, where $\lambda=0.1$

8. (a) Explain Gauss elimination method and point out pivotal elements.
   (b) Use Gauss-Seidel method to construct the solution of the following system.

$$27x + 6y - z = 85, \quad 6x+15y+2z=72, \; x+y+54z=110.$$

9. (a) Apply Gauss Elimination method to find the solution of the system

$$27x + 6y - z = 85, \quad 6x+15y+2z=72, \; x+y+54z=110.$$

   (b) Describe the analytic method for finding the roots of an equation based on Rolle's Theorem.

10. (a) Evaluate $\sqrt{12}$ by applying Newton-Raphson's formula to four places of decimals.
(b) Explain Graeffe's Root squaring method.

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